New and Old Approaches to Ice Sheet Modeling: Solid Earth Geophysics and the Cryosphere

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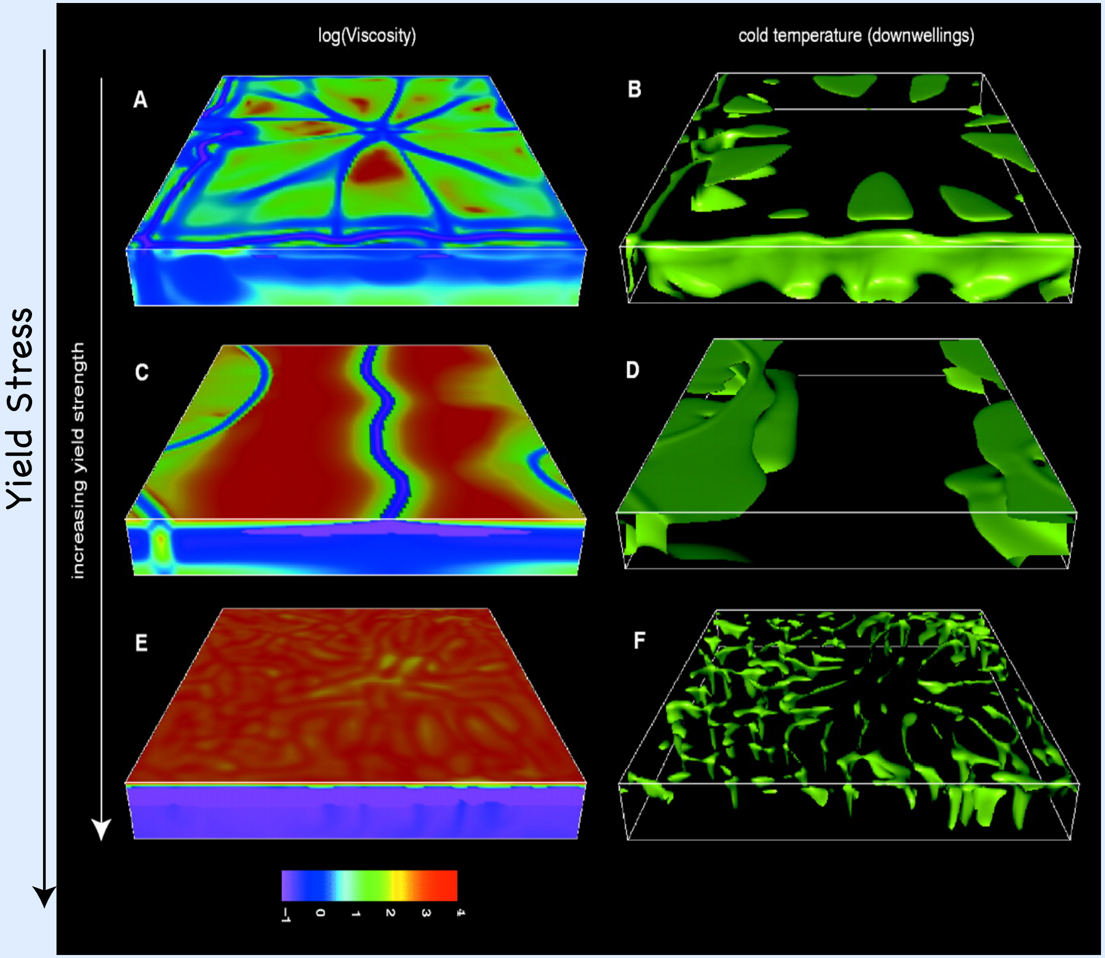
Solid earth geophysics and ice sheets

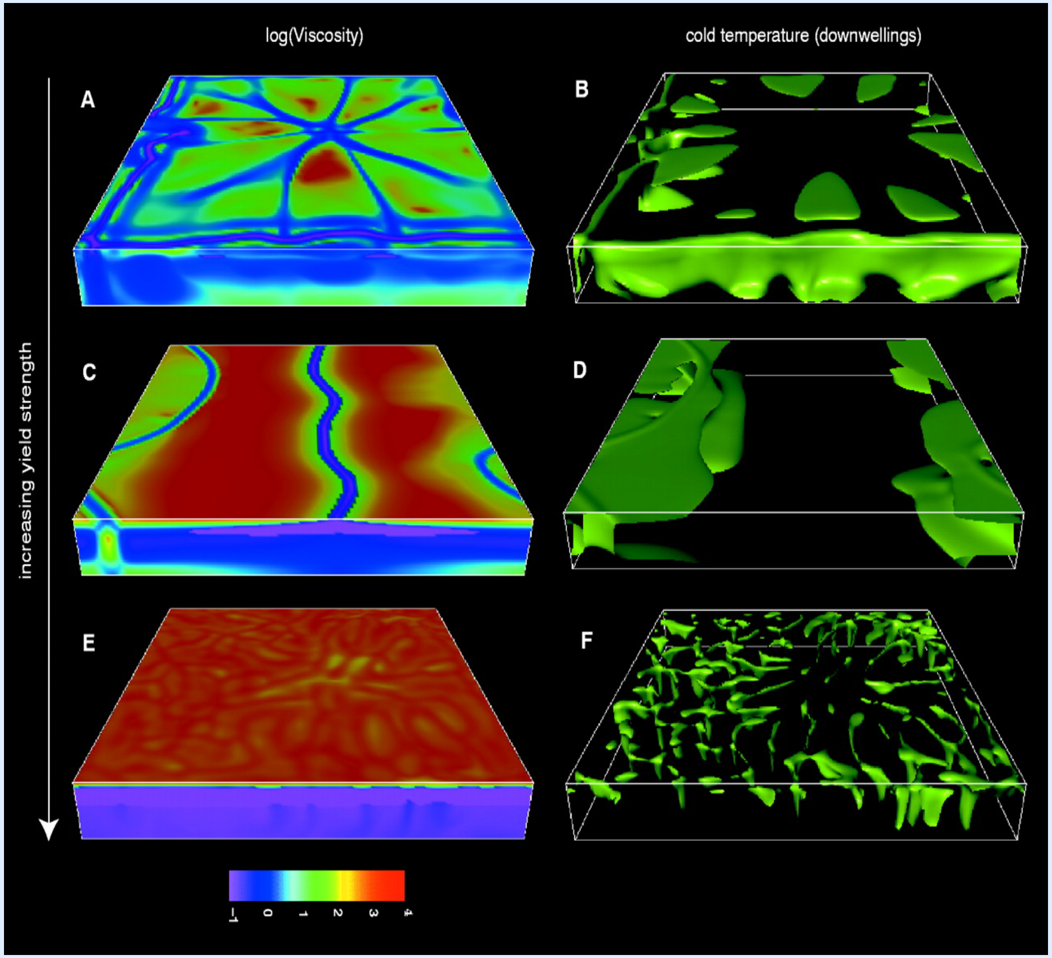
Mantle Convection



Geodynamics and ice sheets:

- Slow viscous flow
- Non-Newtonian fluid
- Temperature dependent viscosity
- Phase changes
- Brittle and ductile flow regimes

Solid earth geophysics and ice sheets



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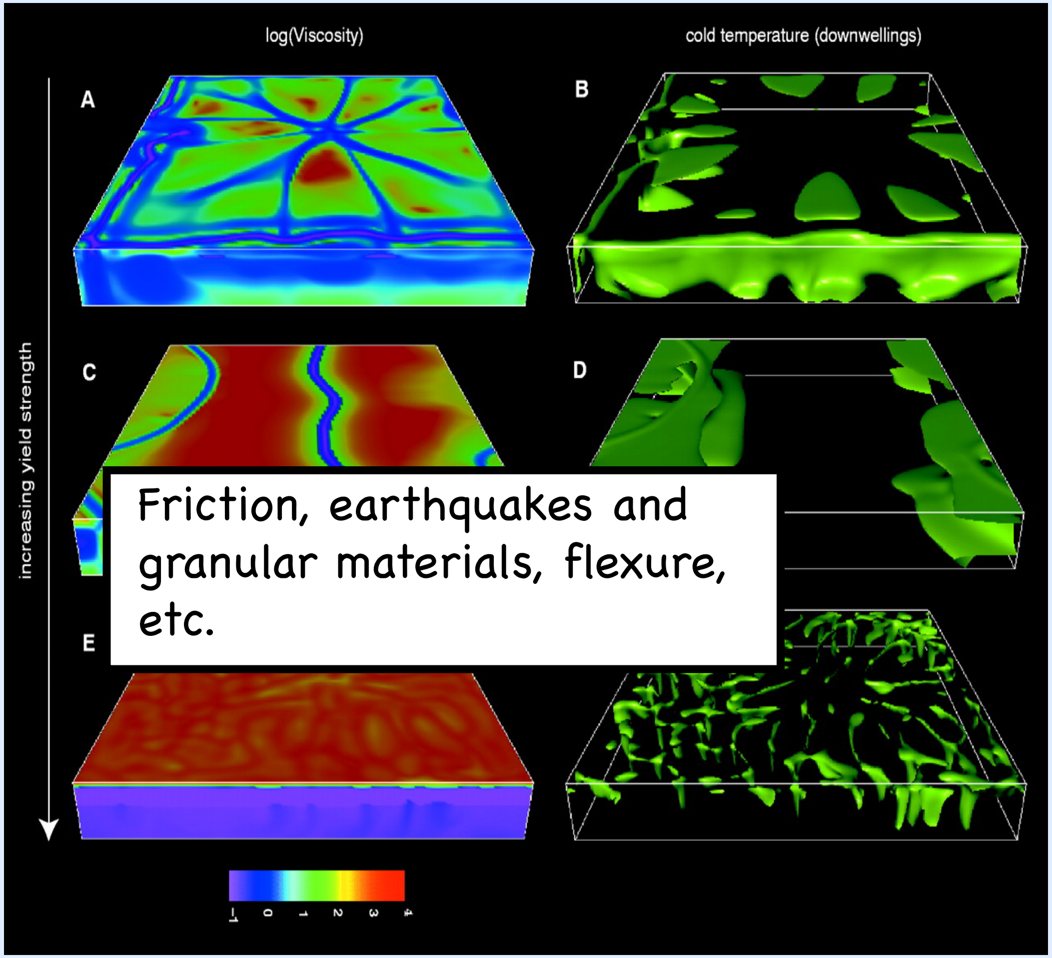
Low Peclet number:

$$Pe = \frac{\text{Mass Diffusion}}{\text{Mass Advection}}$$

Key differences:

- Ice sheets (nearly) barotropic
- Ice sheets don't conserve mass (mass added and removed)

Solid earth geophysics and ice sheets



Friction, earthquakes and granular materials, flexure, etc.

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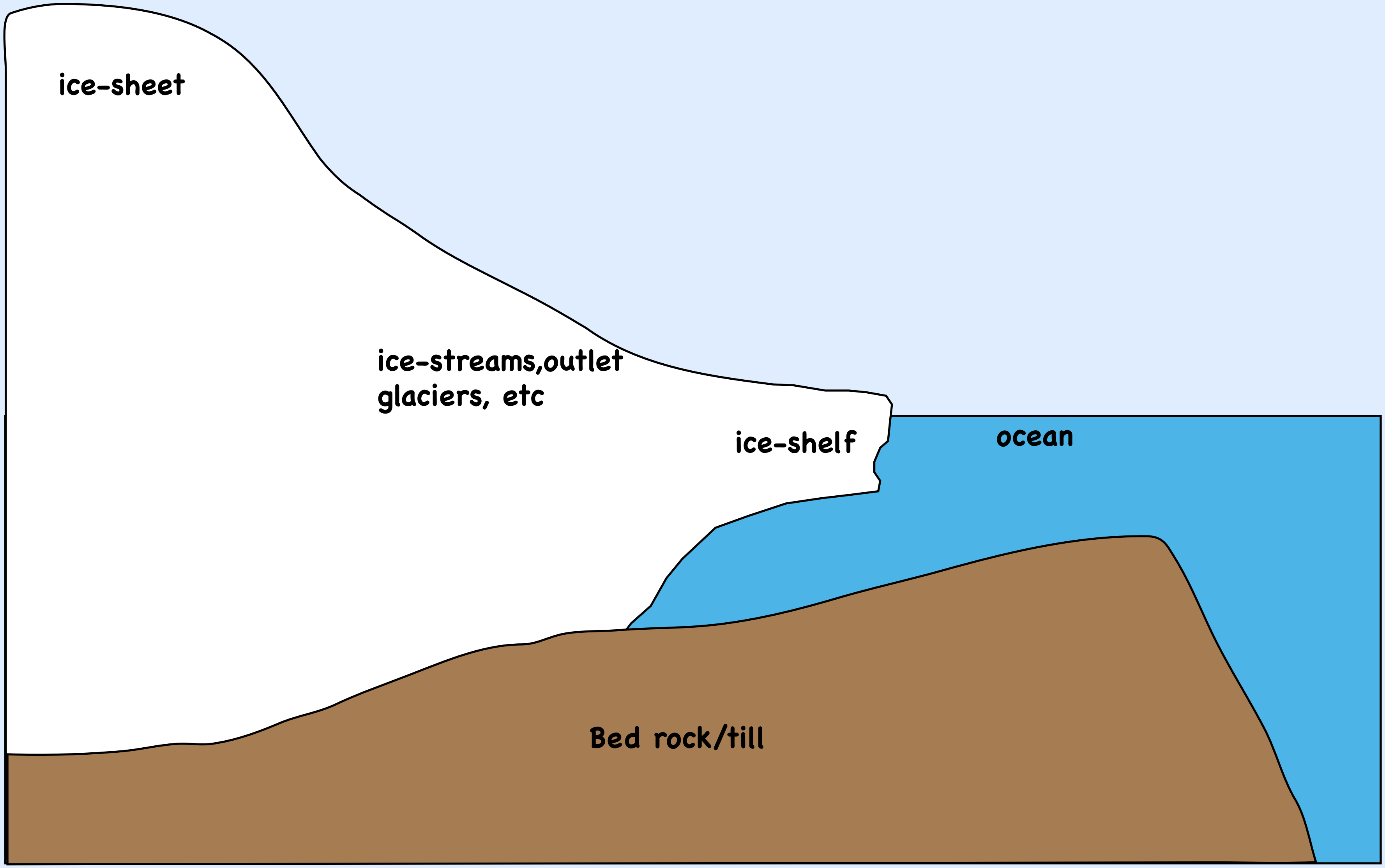

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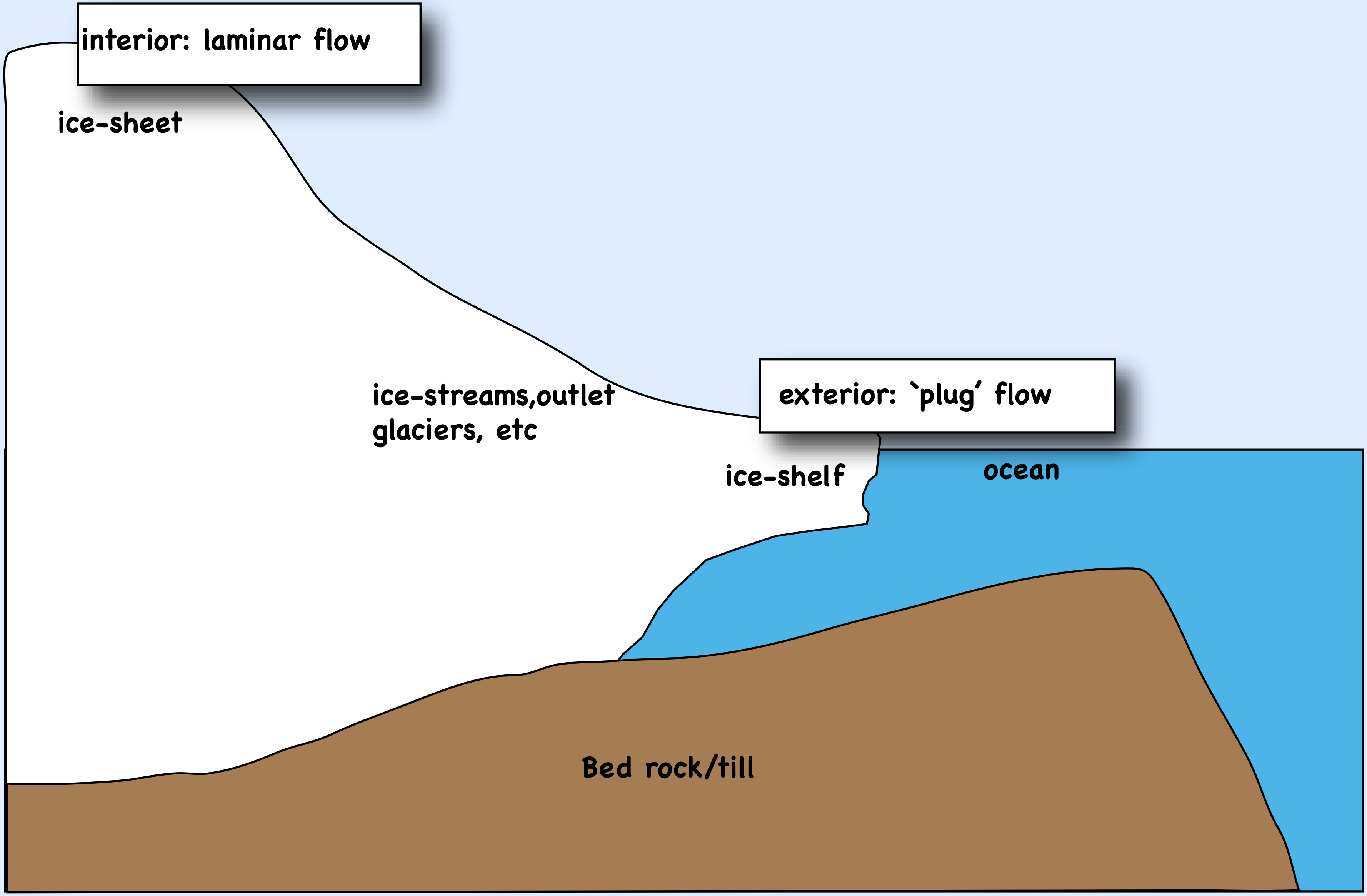

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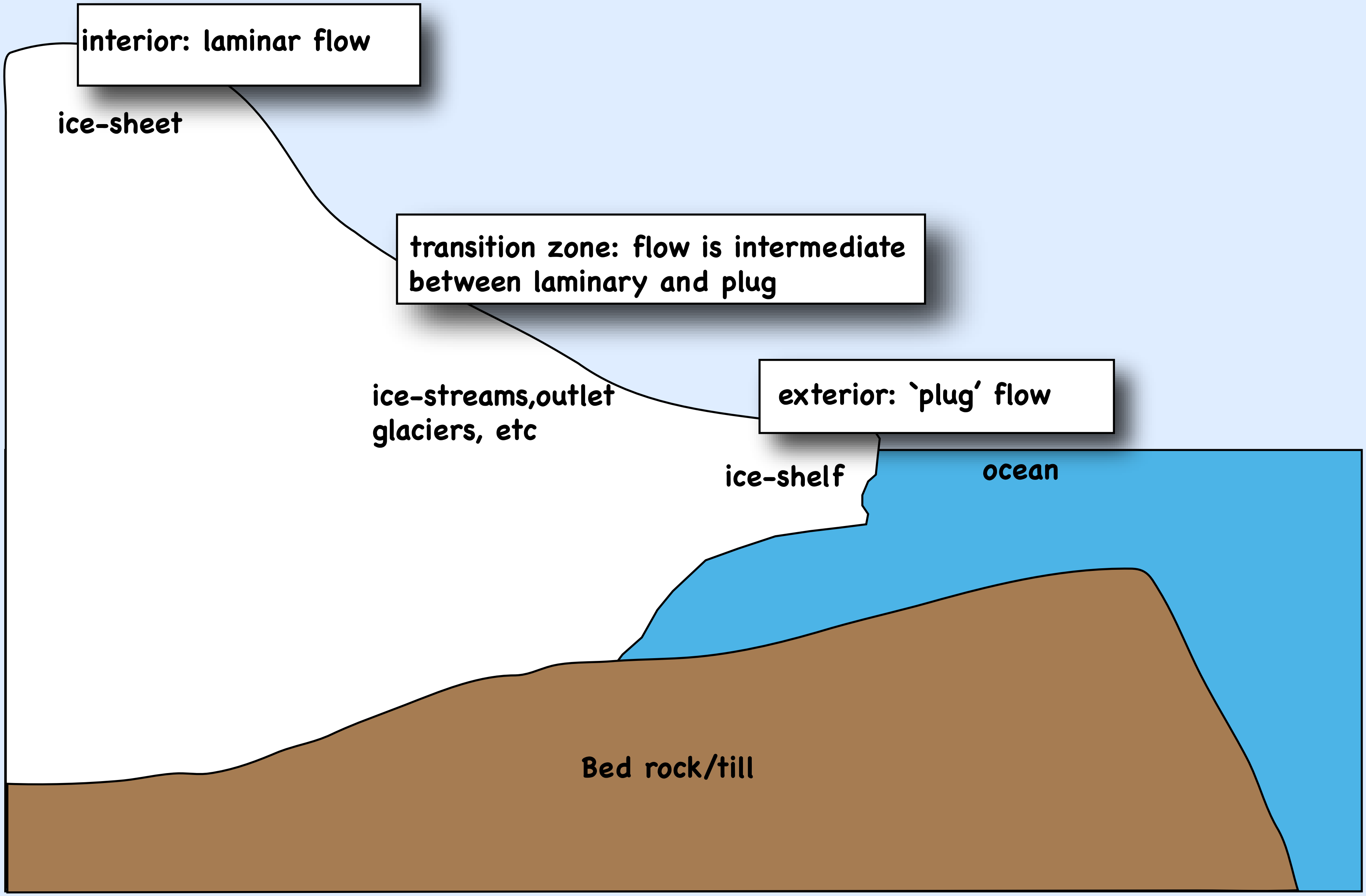


Ice sheet dynamics

Shallow-ice approximation (SIA):

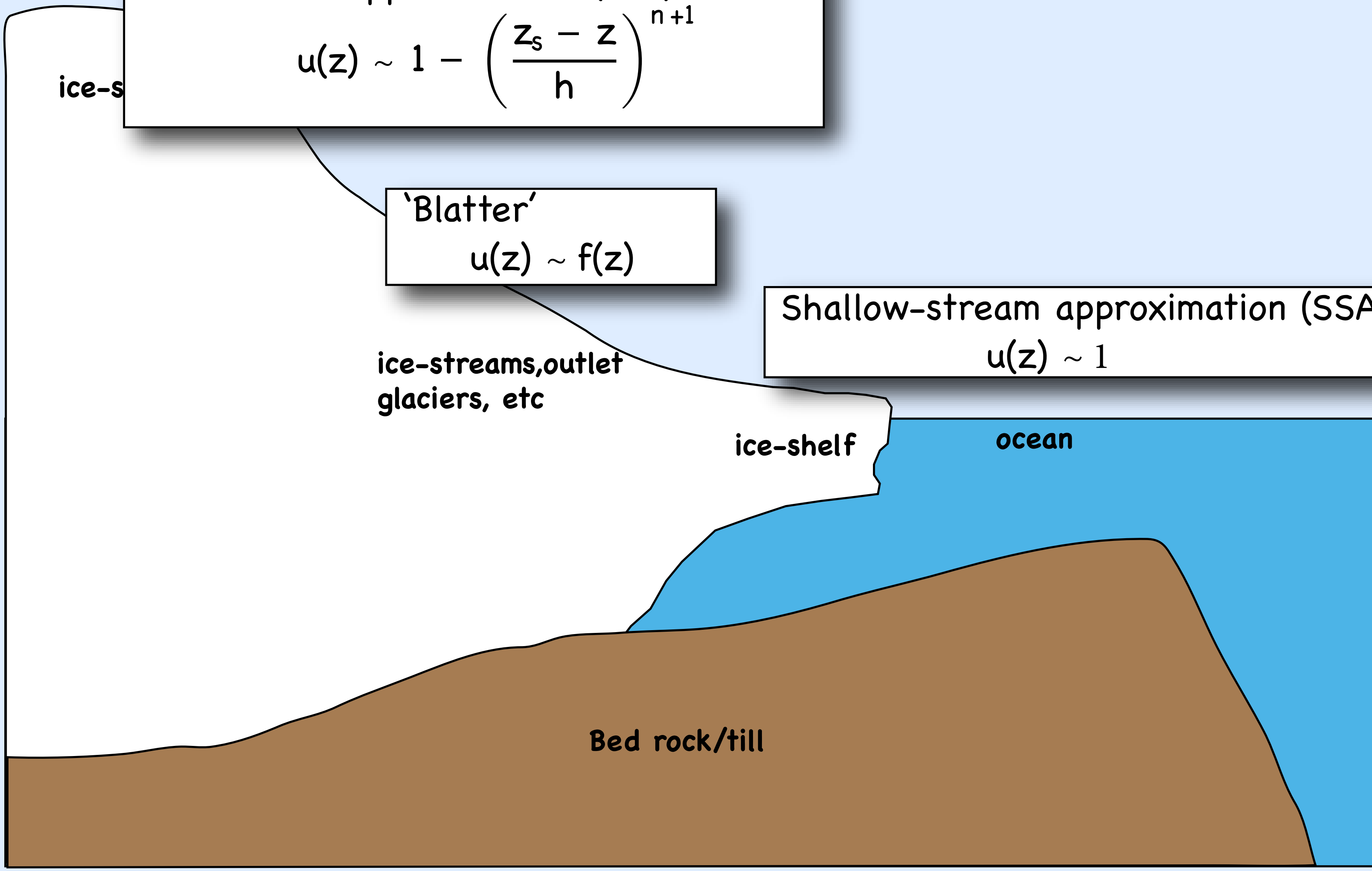
$$u(z) \sim 1 - \left(\frac{z_s - z}{h}\right)^{n+1}$$

'Blatter'

$$u(z) \sim f(z)$$

Shallow-stream approximation (SSA):

$$u(z) \sim 1$$



Ice sheet dynamics

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$$u(z) \sim 1 - \left(\frac{z_s - z}{h}\right)^{n+1}$$

Goal: Describe transition from SIA to SSA

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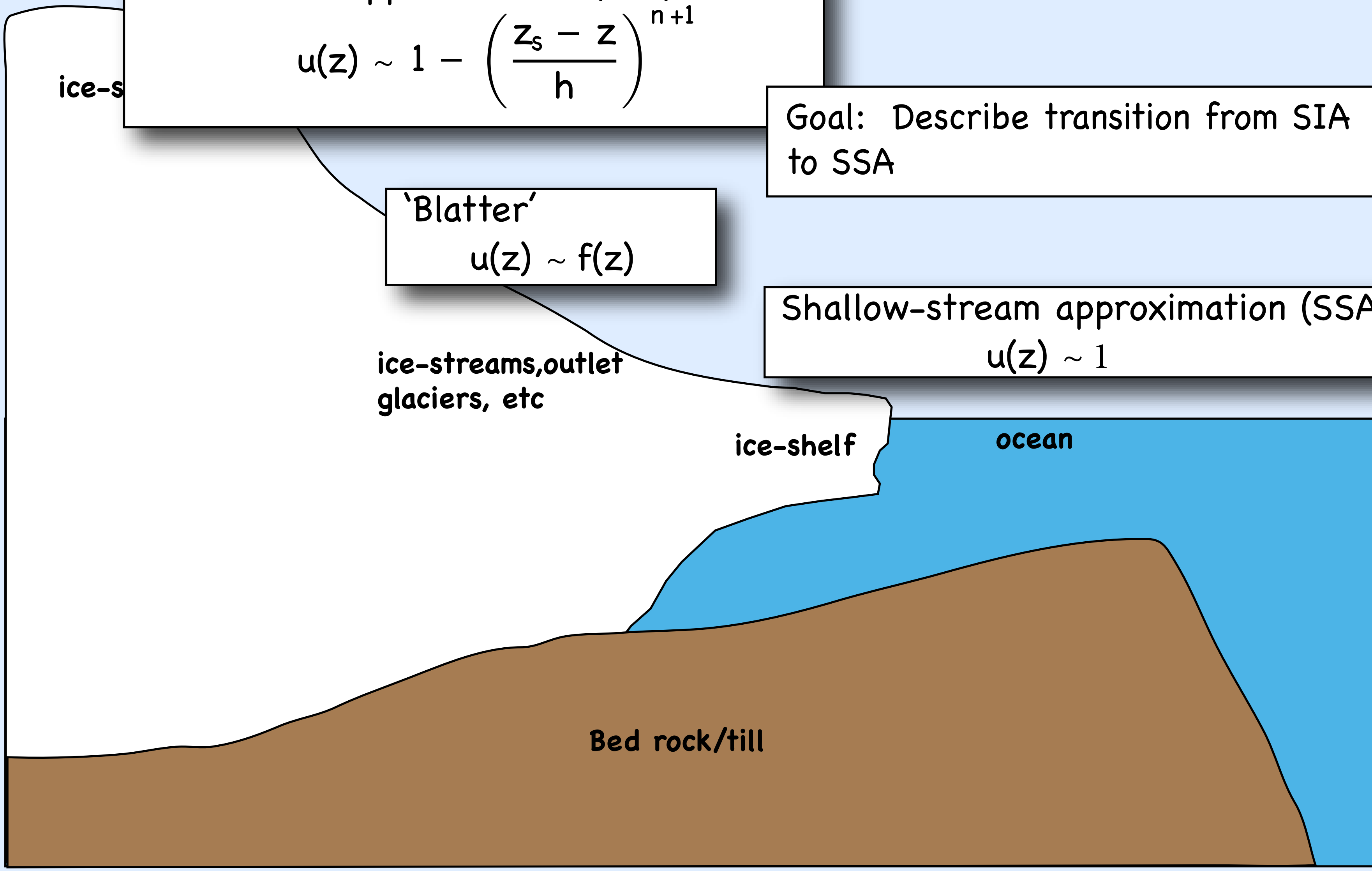


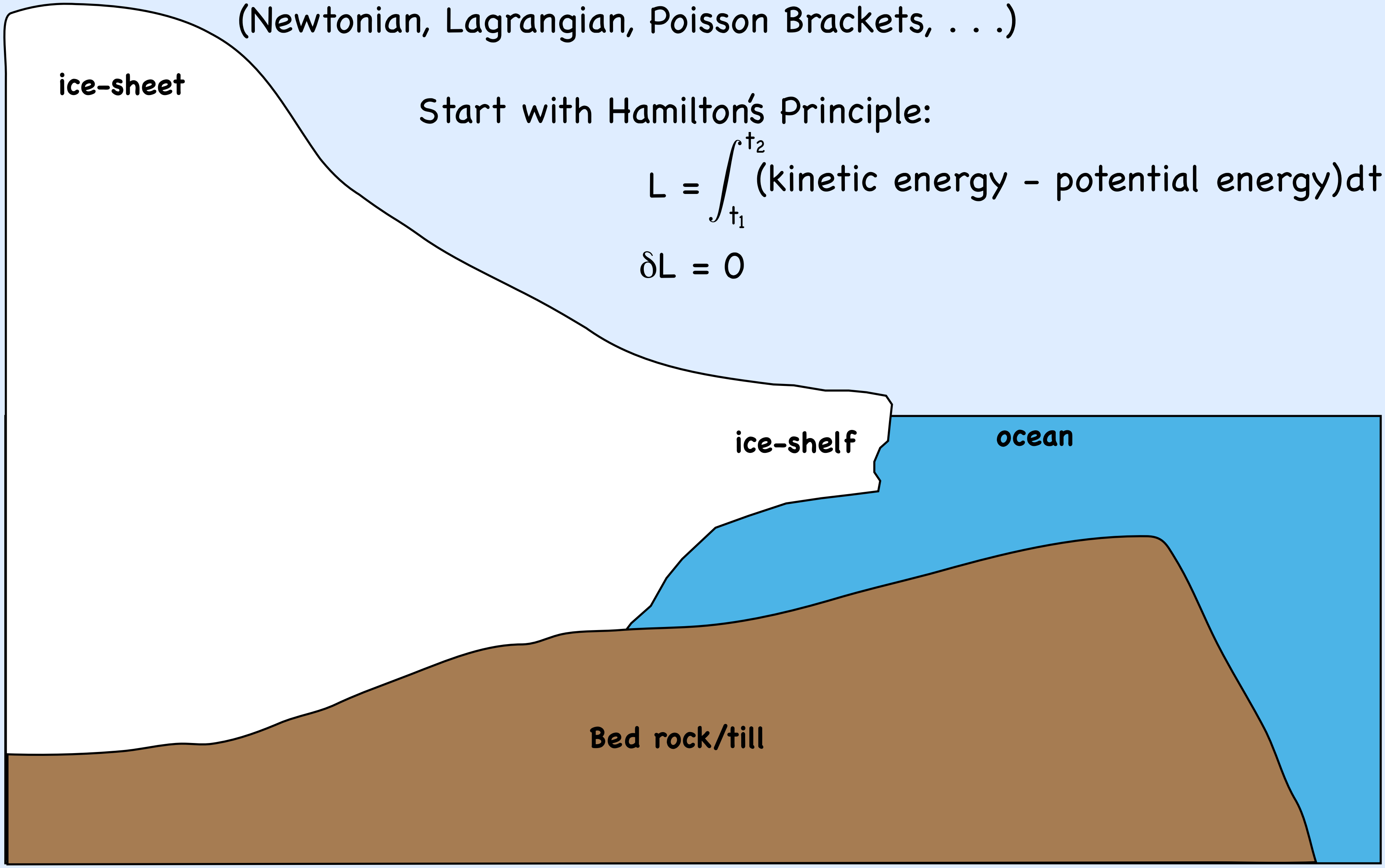
A simple approximation

Multiple (equivalent) approaches to mechanics:
(Newtonian, Lagrangian, Poisson Brackets, . . .)

Start with Hamilton's Principle:

$$L = \int_{t_1}^{t_2} (\text{kinetic energy} - \text{potential energy})\,dt$$

$$\delta L = 0$$



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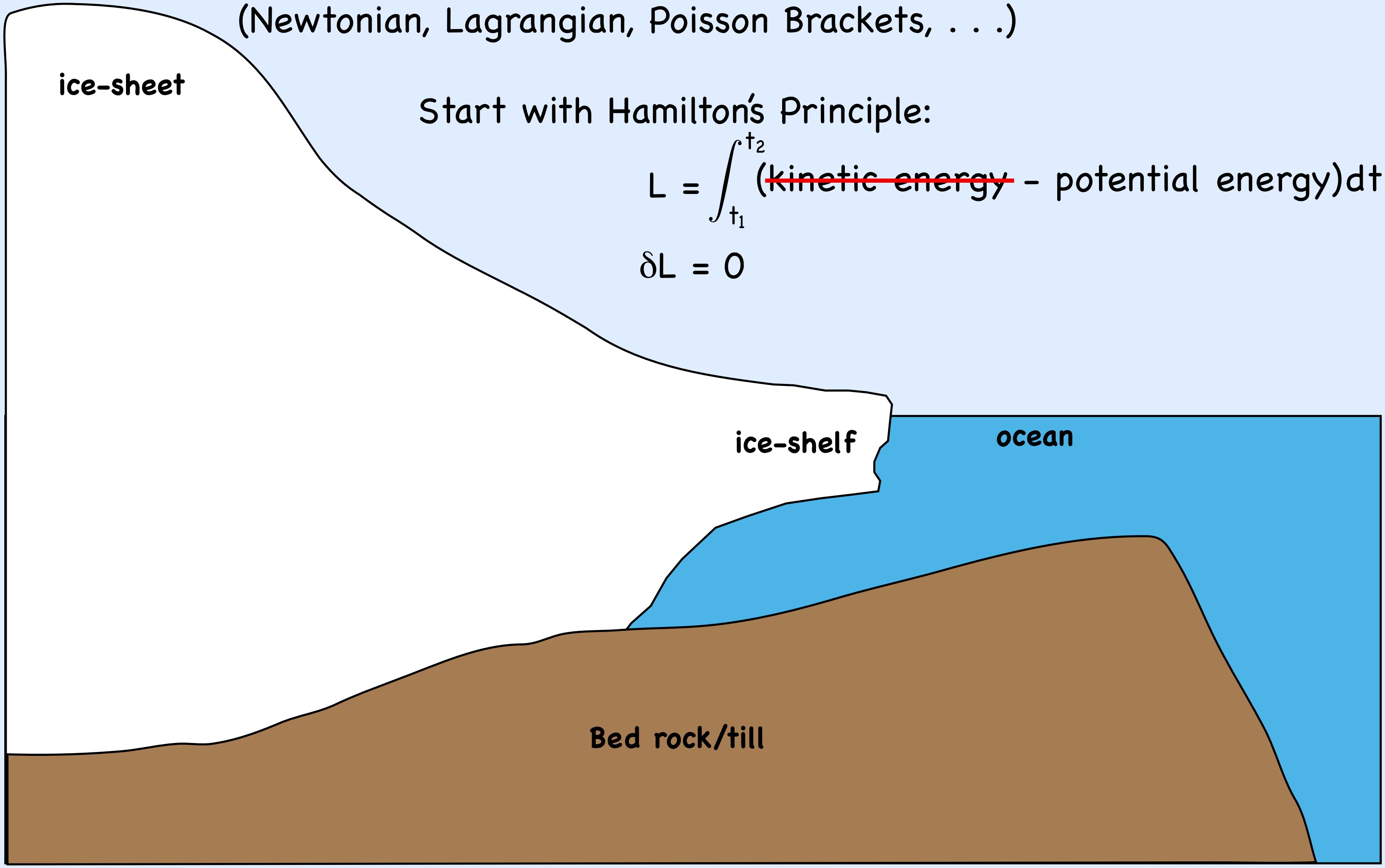

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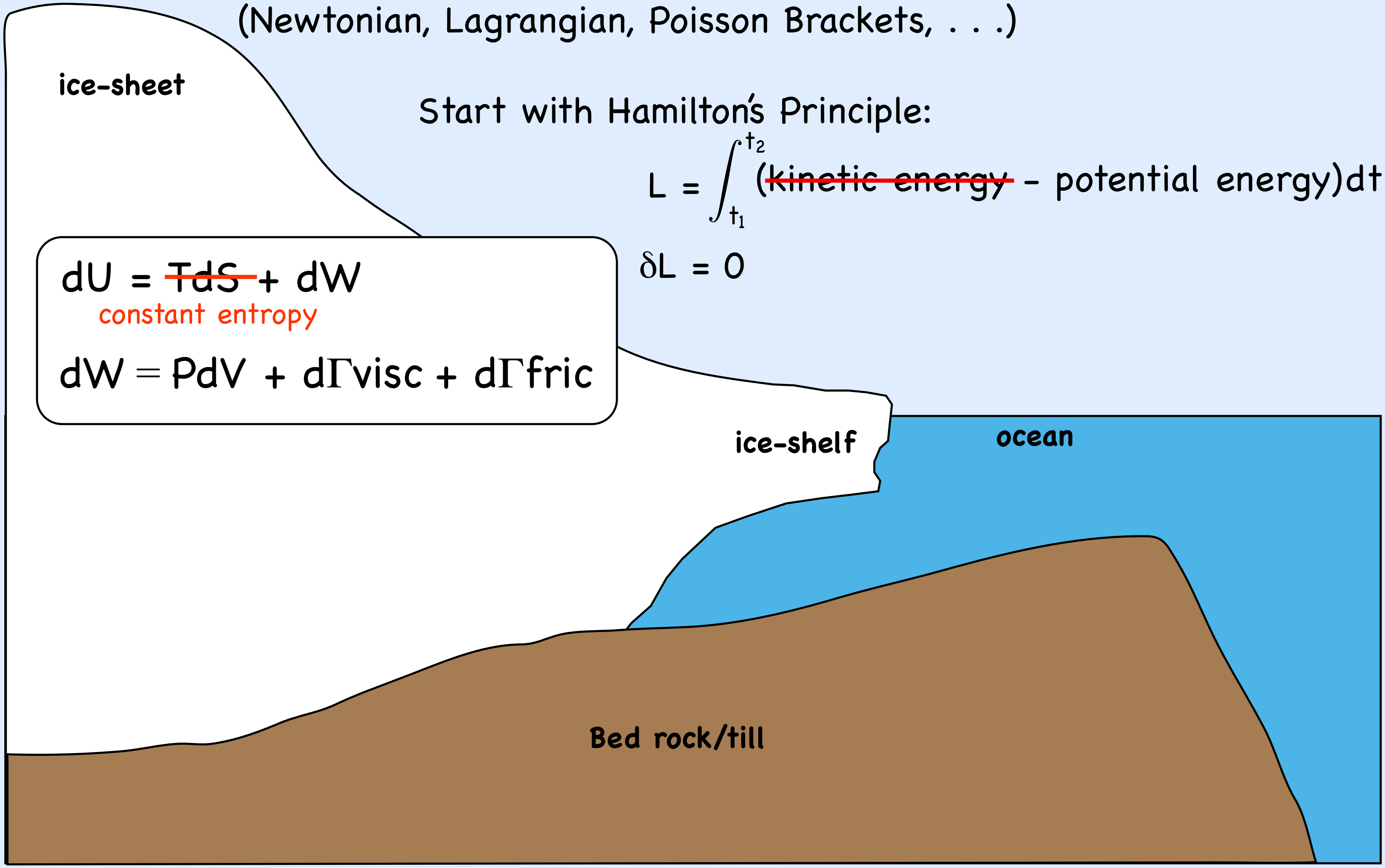
constant entropy

$$dW = PdV + d\Gamma visc + d\Gamma fric$$



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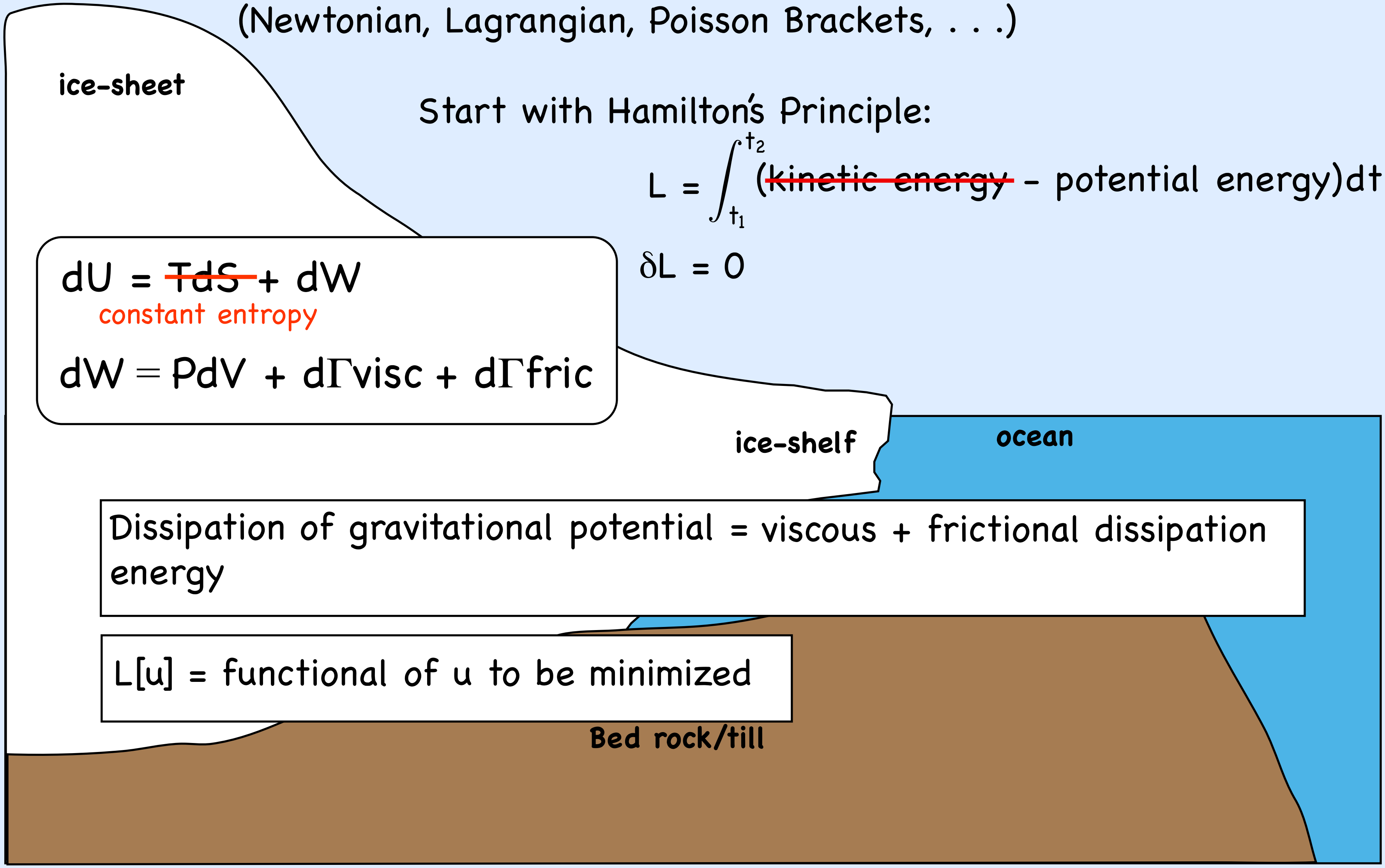
Dissipation of gravitational potential = viscous + frictional dissipation energy

L[u] = functional of u to be minimized



A simple approximation: Rayleigh-Ritz Method

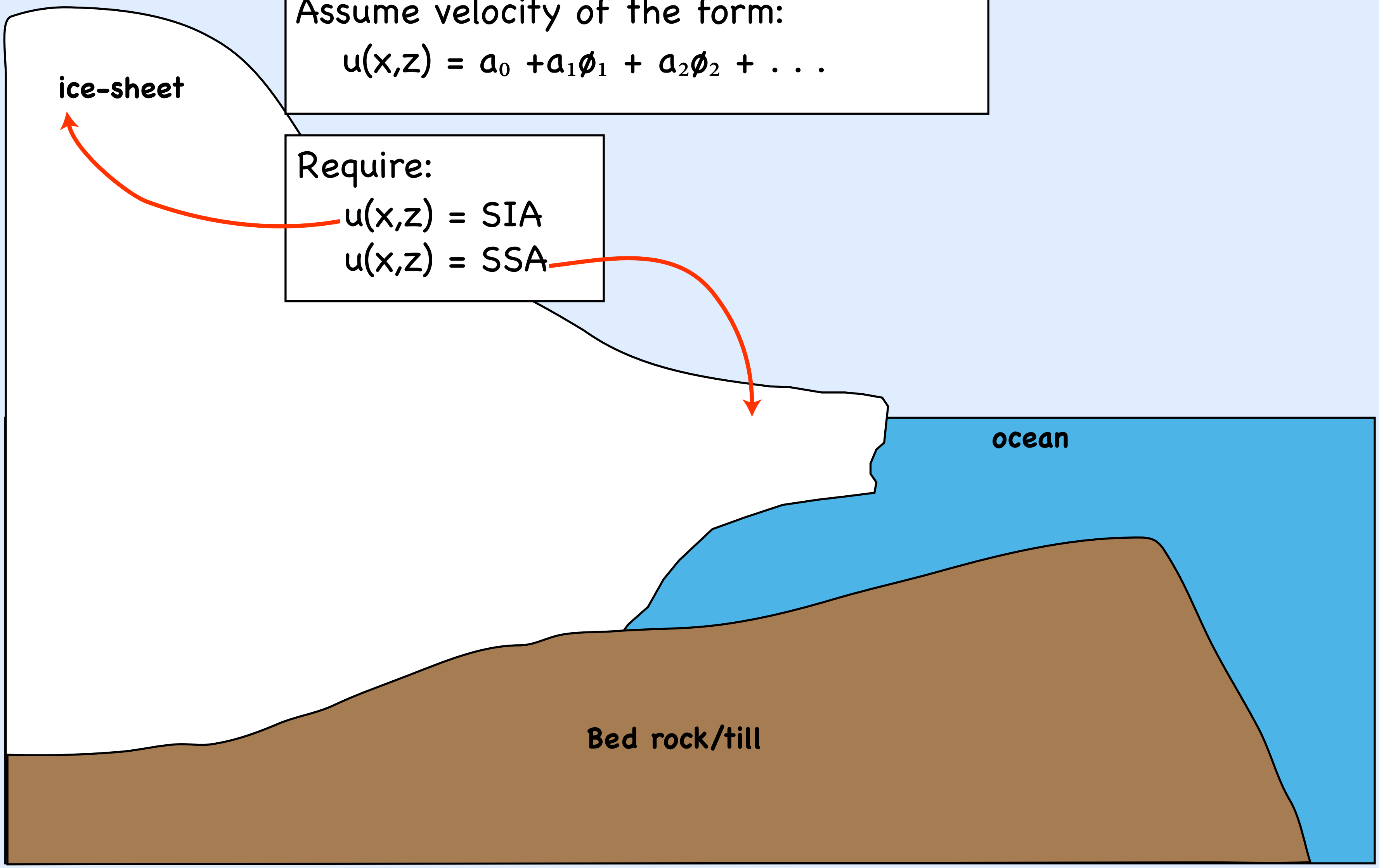
Assume velocity of the form:

$$u(x,z) = a_0 + a_1\phi_1 + a_2\phi_2 + \ldots$$

Require:

$u(x,z)$ = SIA

$u(x,z)$ = SSA



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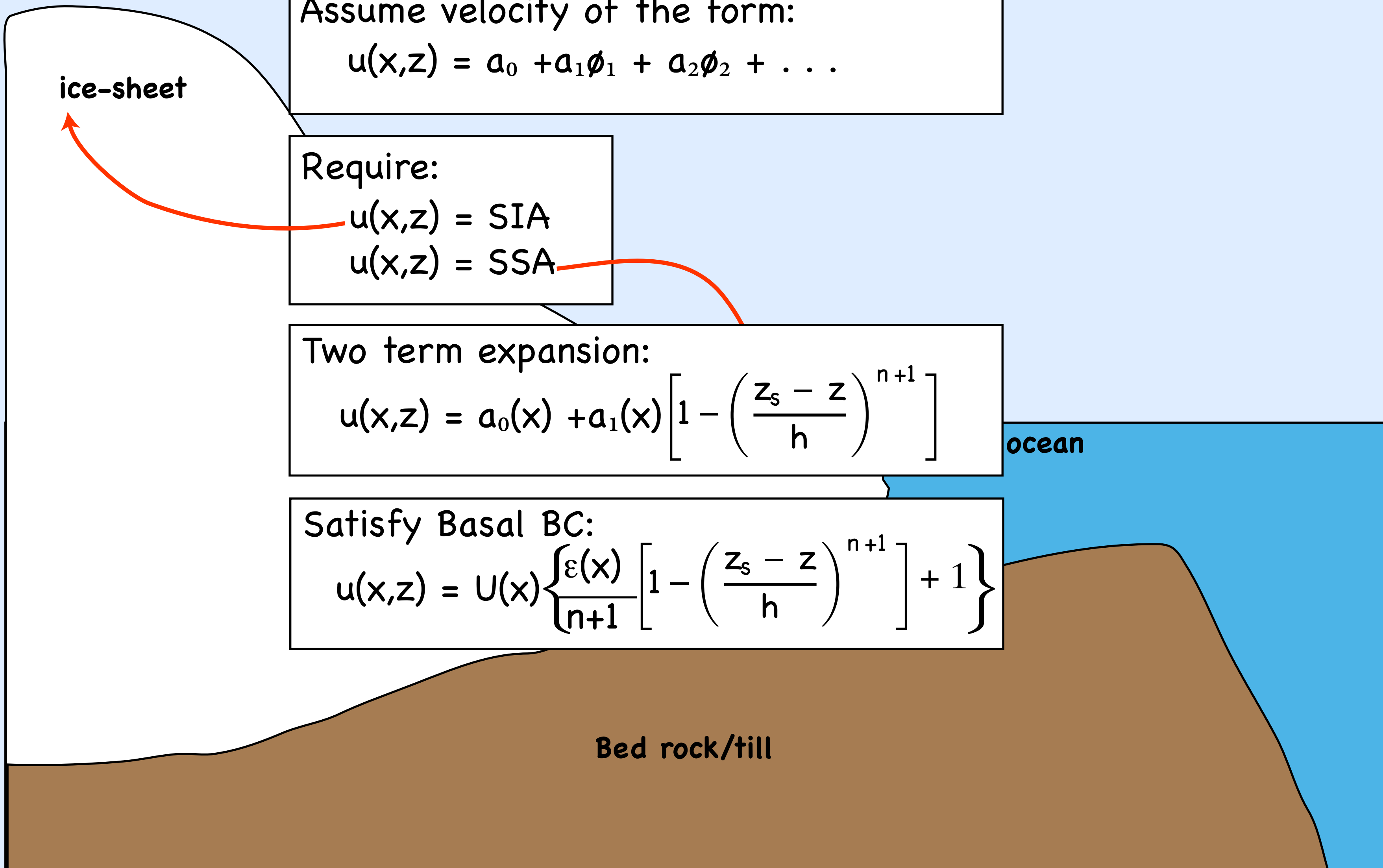

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Two term expansion:

$$u(x,z) = a_0(x) + a_1(x)\left[1 - \left(\frac{z_s - z}{h}\right)^{n+1}\right]$$

Satisfy Basal BC:

$$u(x,z) = U(x)\left\{\frac{\varepsilon(x)}{n+1}\left[1 - \left(\frac{z_s - z}{h}\right)^{n+1}\right] + 1\right\}$$

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$$\varepsilon(x) = \frac{\beta h}{\mu}$$

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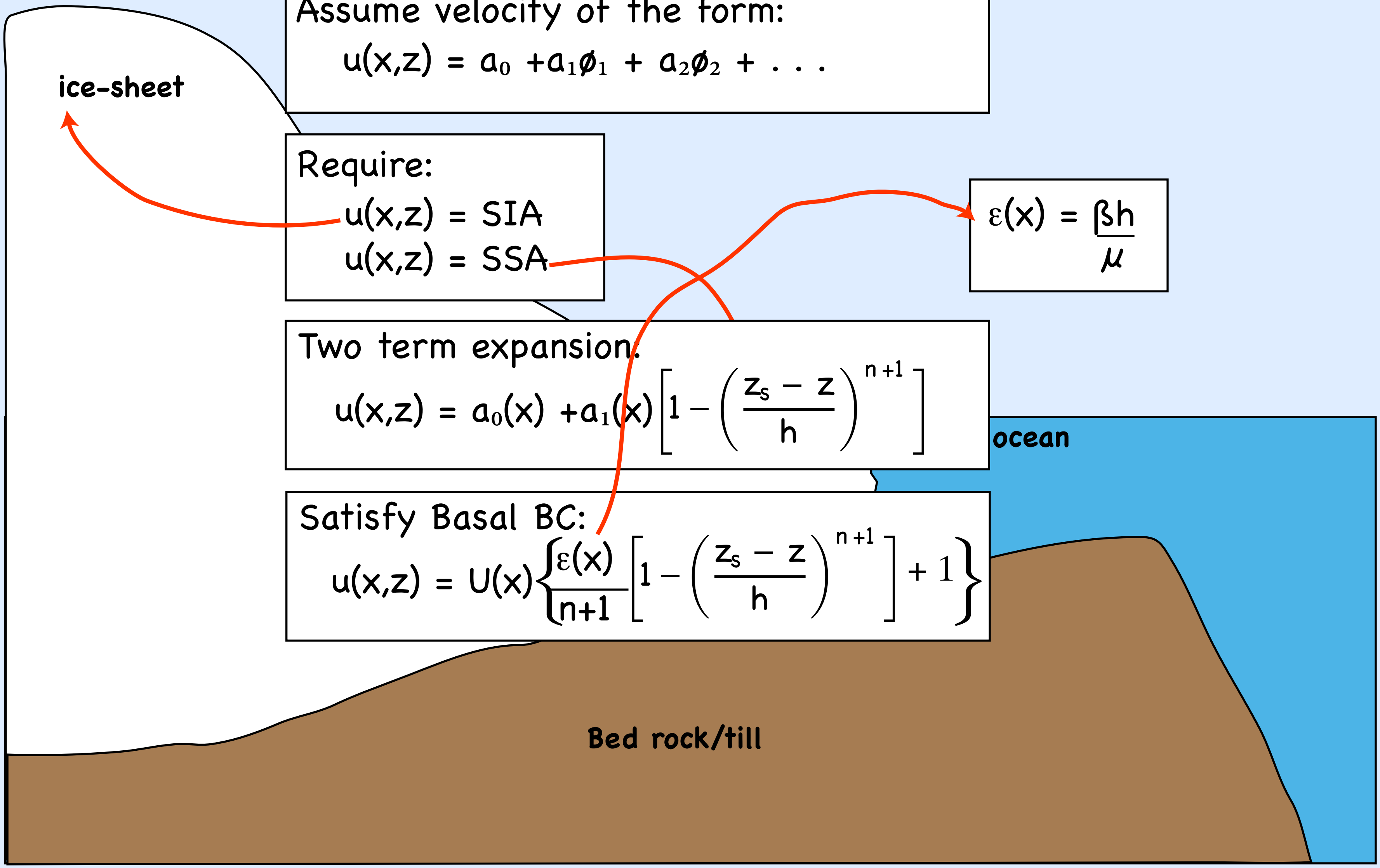


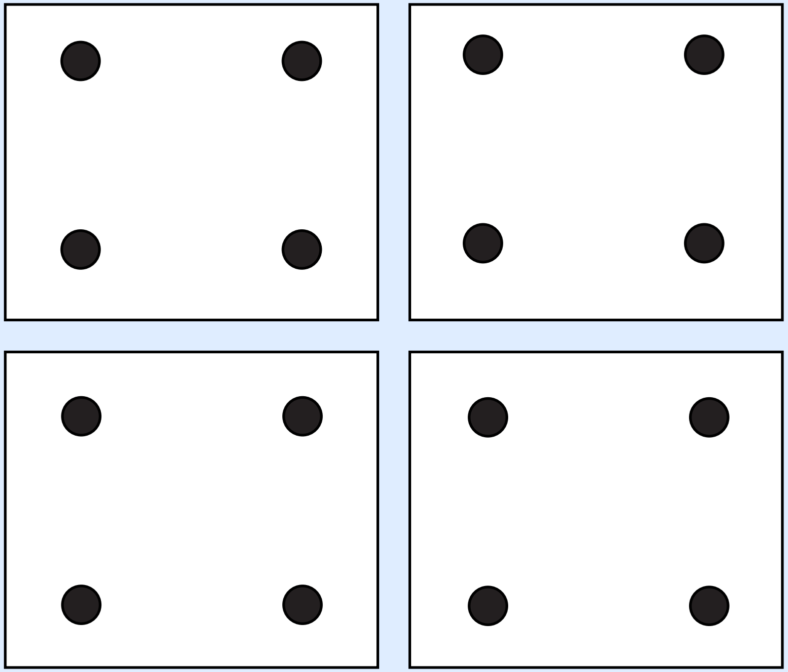
(Non-dimensional) Diagnostic and Prognostic Equations

$$\frac{\partial}{\partial x}\left[c_1(x)\frac{\partial U}{\partial x}\right] + c_2(x)U = h\frac{\partial z_s}{\partial x} + \epsilon\delta^{-2}\beta' U$$

Transition from SIA to SSA introduces 'quasi-boundary layers'

$$\frac{\partial h}{\partial t} + \underbrace{\frac{\partial}{\partial x}\left[h\left(u - u_{SIA}\right)\right]}_{\text{Non-shallow ice flux}} - \underbrace{\frac{\partial}{\partial x}\left(D\frac{\partial h}{\partial x}\right)}_{\text{shallow ice flux}} = M$$

Time Integration: Mixed Eulerian-Lagrangian Methods

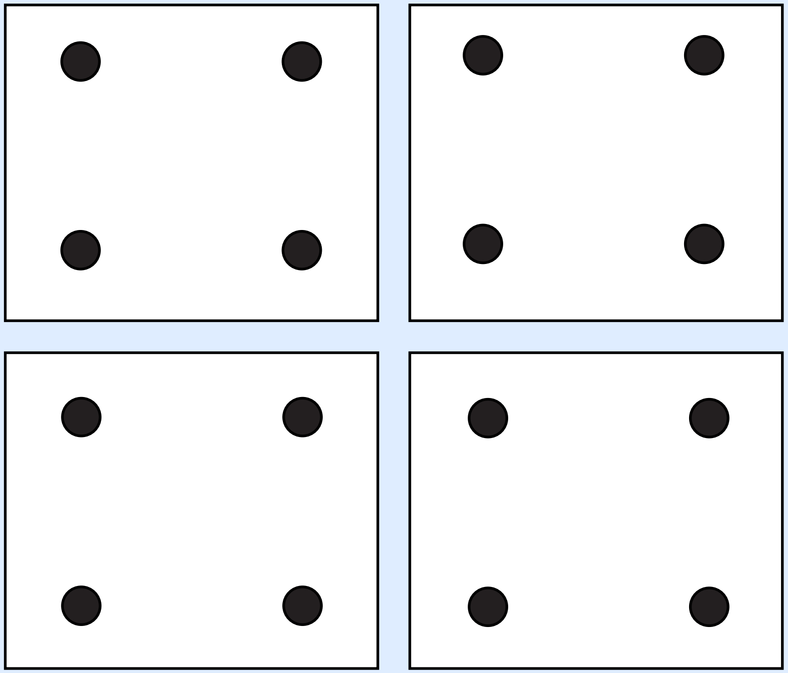


Time Integration: Mixed Eulerian-Lagrangian Methods

Allow integration points to advect as Lagrangian tracers

Create/destroy tracers to preserve accuracy

Can view tracers as discrete mass - **mass is conserved exactly**



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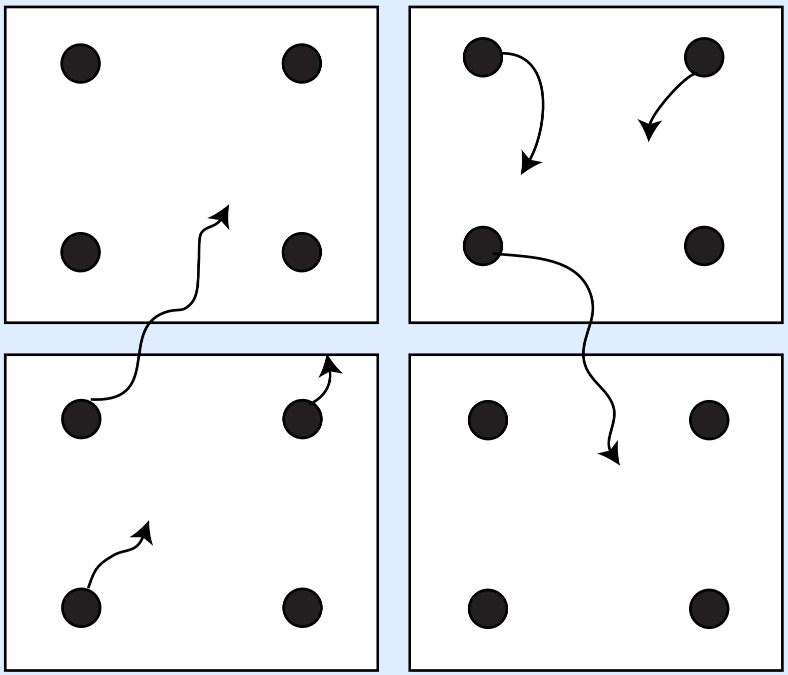

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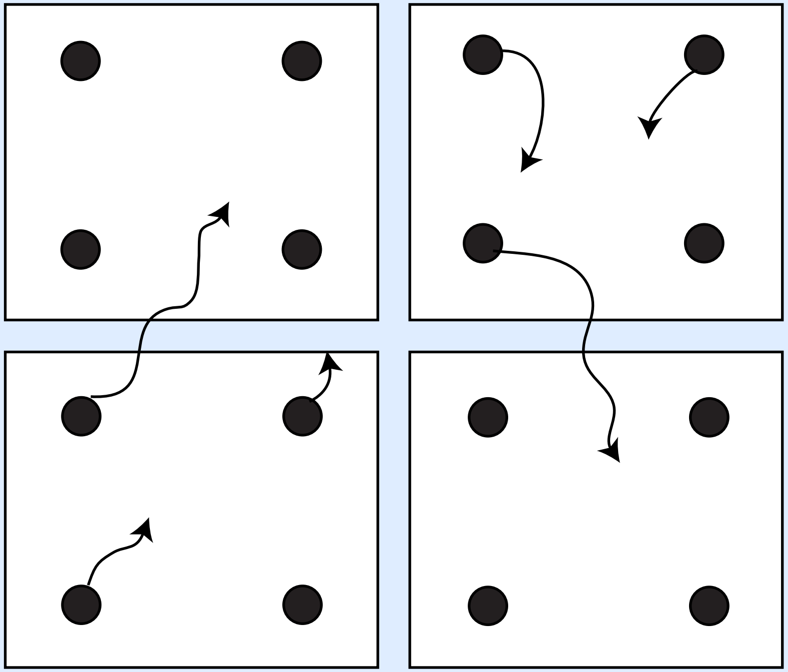


nodes migrate

Advantages:

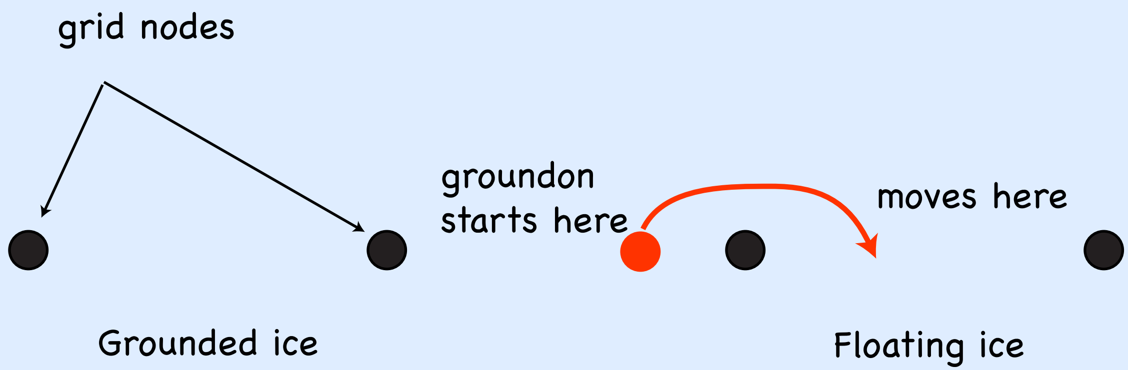
Lagrangian advection of information (no fake diffusion)
Re-meshing is intepolation free (nodes don't get moved)
Easy to deal with evolving free surfaces

Disadvantages:

Resolution migrates
Ad-hoc rules to add/subtract nodal points
Boundary conditions can be problematic
Need about 2x as many grid nodes

Grounding line migration: 'Particle Method'

- Represent grounding line as a quasi-particle - a 'groundon'
- Grounding line migration - find groundon trajectory



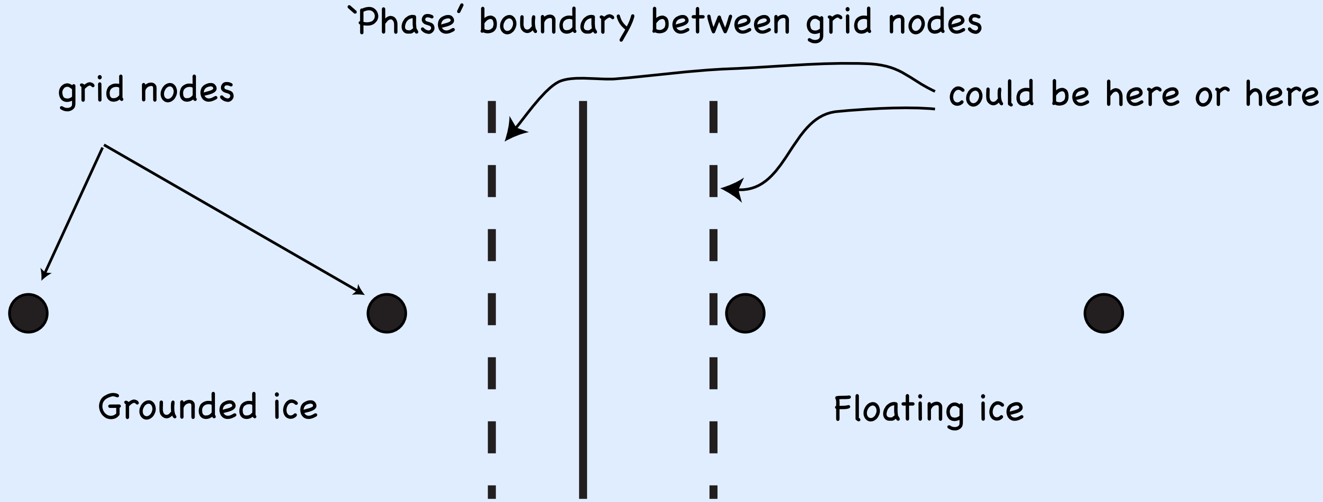
- Evolution equation for each groundon:

$$\frac{dg_1}{dt} = f_1(t)$$

- Solve simultaneously with advection-diffusion eqn.

Grounding line migration: 'Enthalpy Method'

- Analogy with diffusion problems with moving phase boundaries
- Consider floating ice and grounded ice to be different phases



Boundary has to advance/retreat one discreet grid point at a time

Introduce parameter f (varies from 0 to 1) = $\frac{\text{Volume of ice in grid}}{\text{Volume to ground entire cell}}$

Track flux of ice into each grid over time - removes some of hysteresis problems

Which ice dynamics approximation is best?

- Depends on question asked **and** data available
- A model is an improvement if it explains more data
- But . . . also need to account for increased number of degrees of freedom

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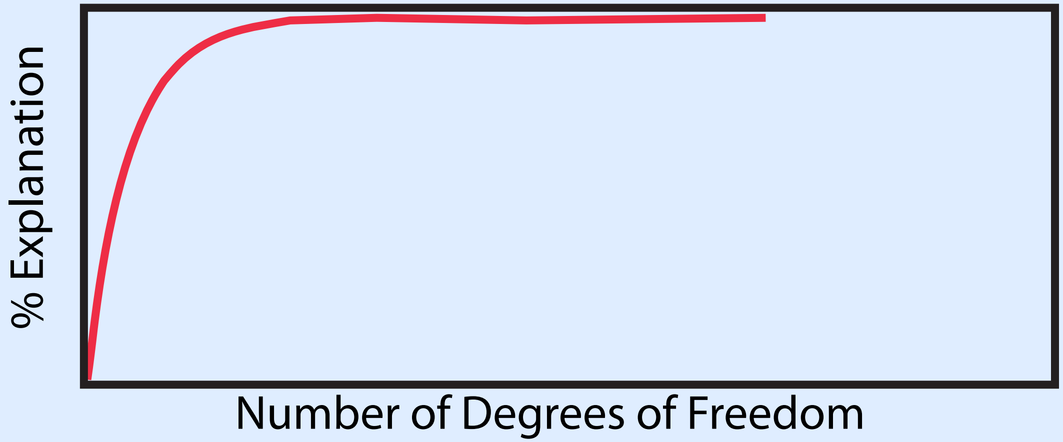

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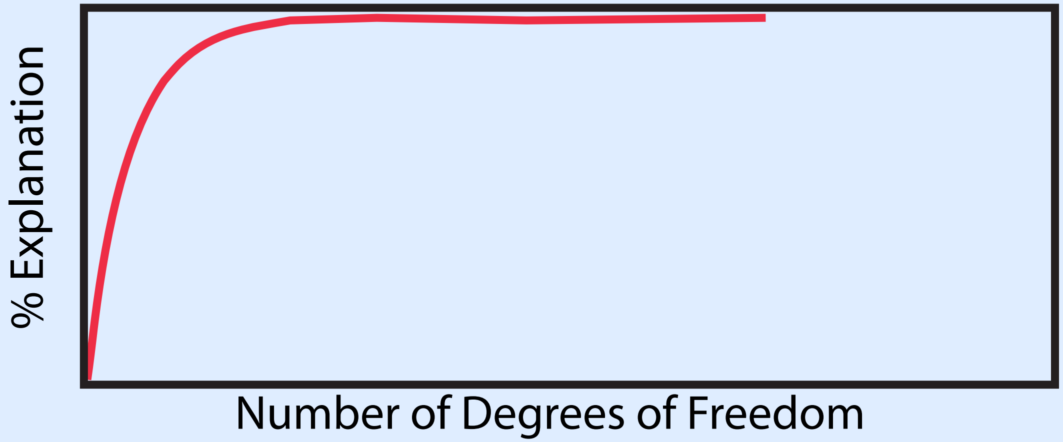

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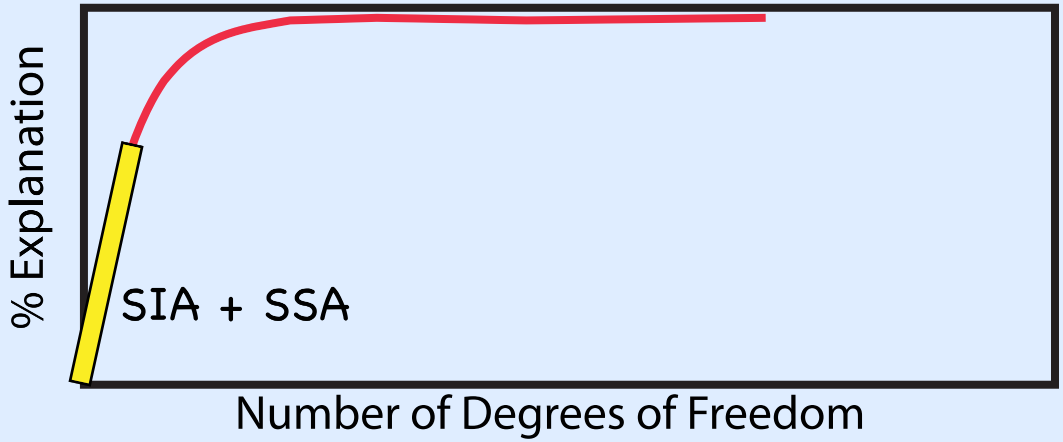

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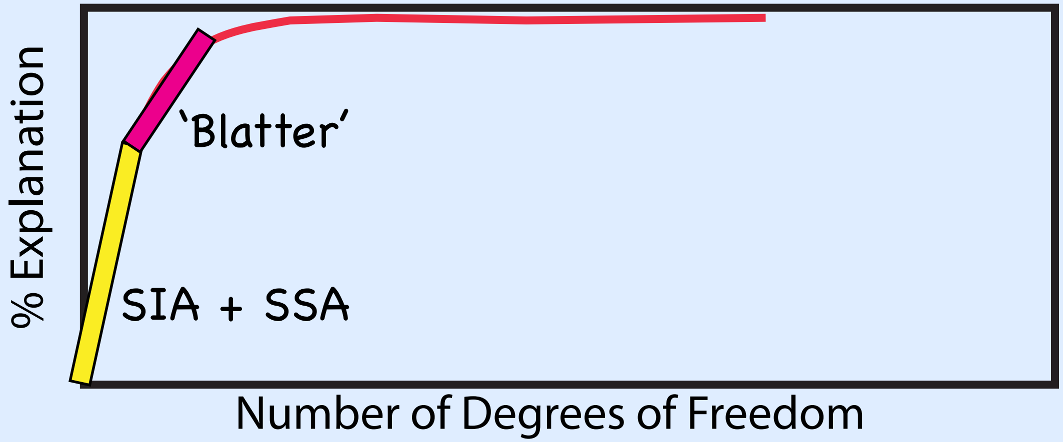

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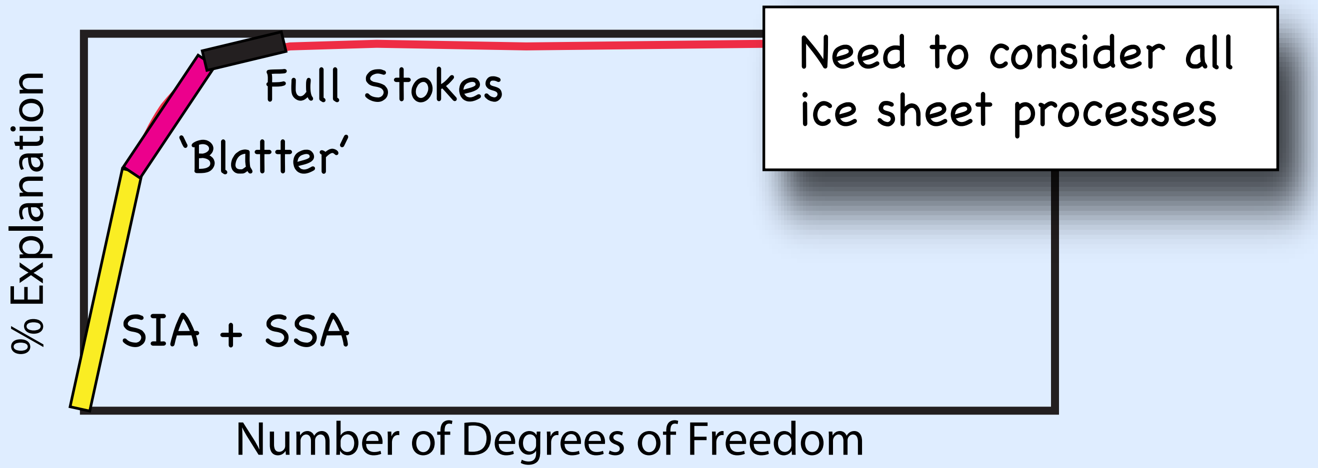

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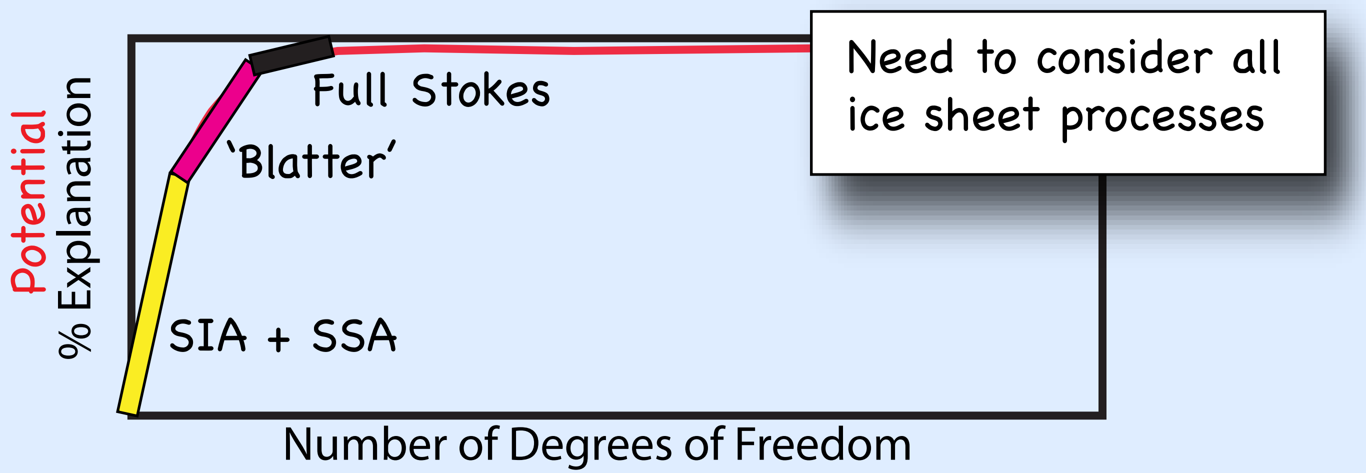

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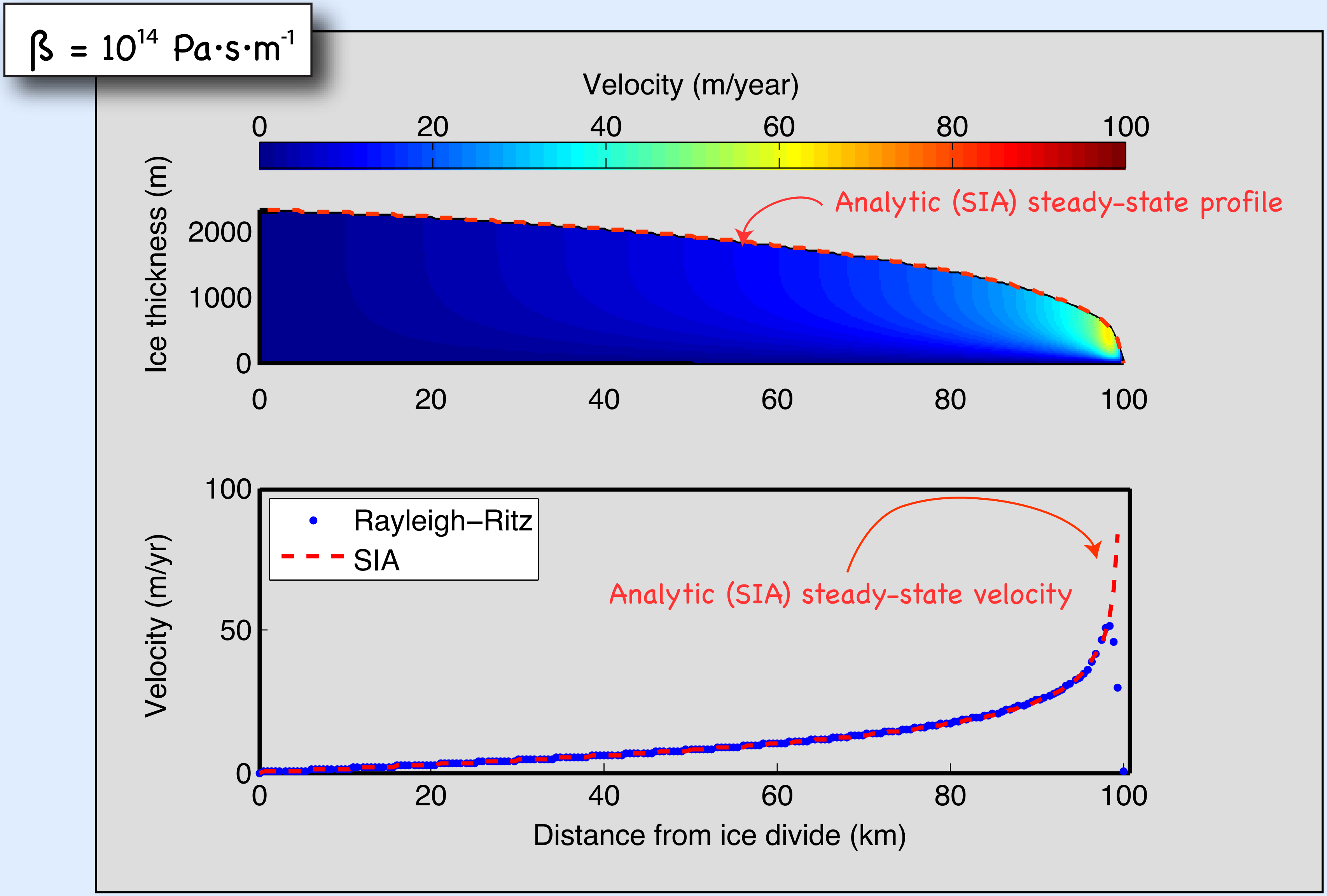

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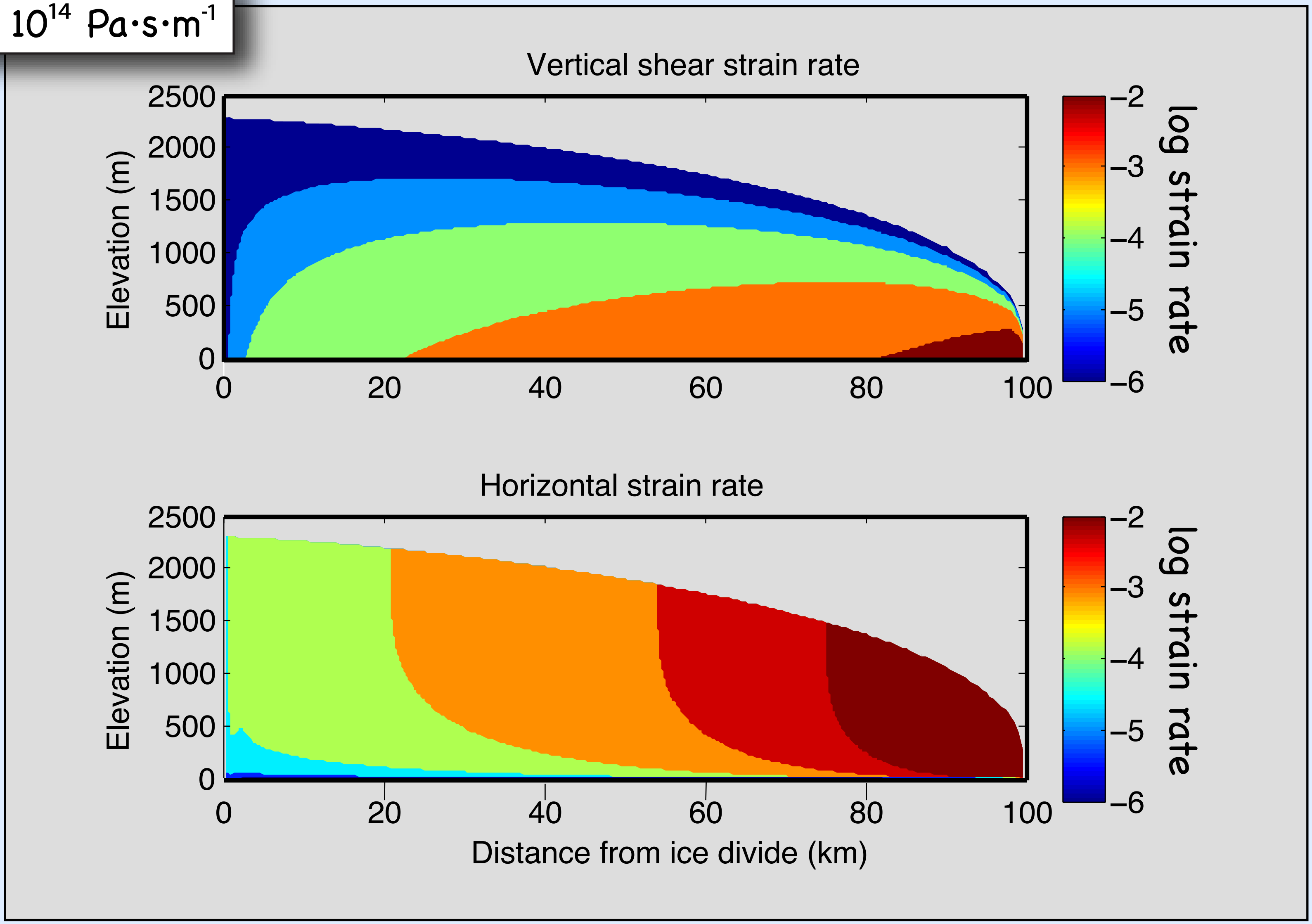
$\beta = 10^{14}\ \mathrm{Pa{\cdot}s{\cdot}m^{-1}}$



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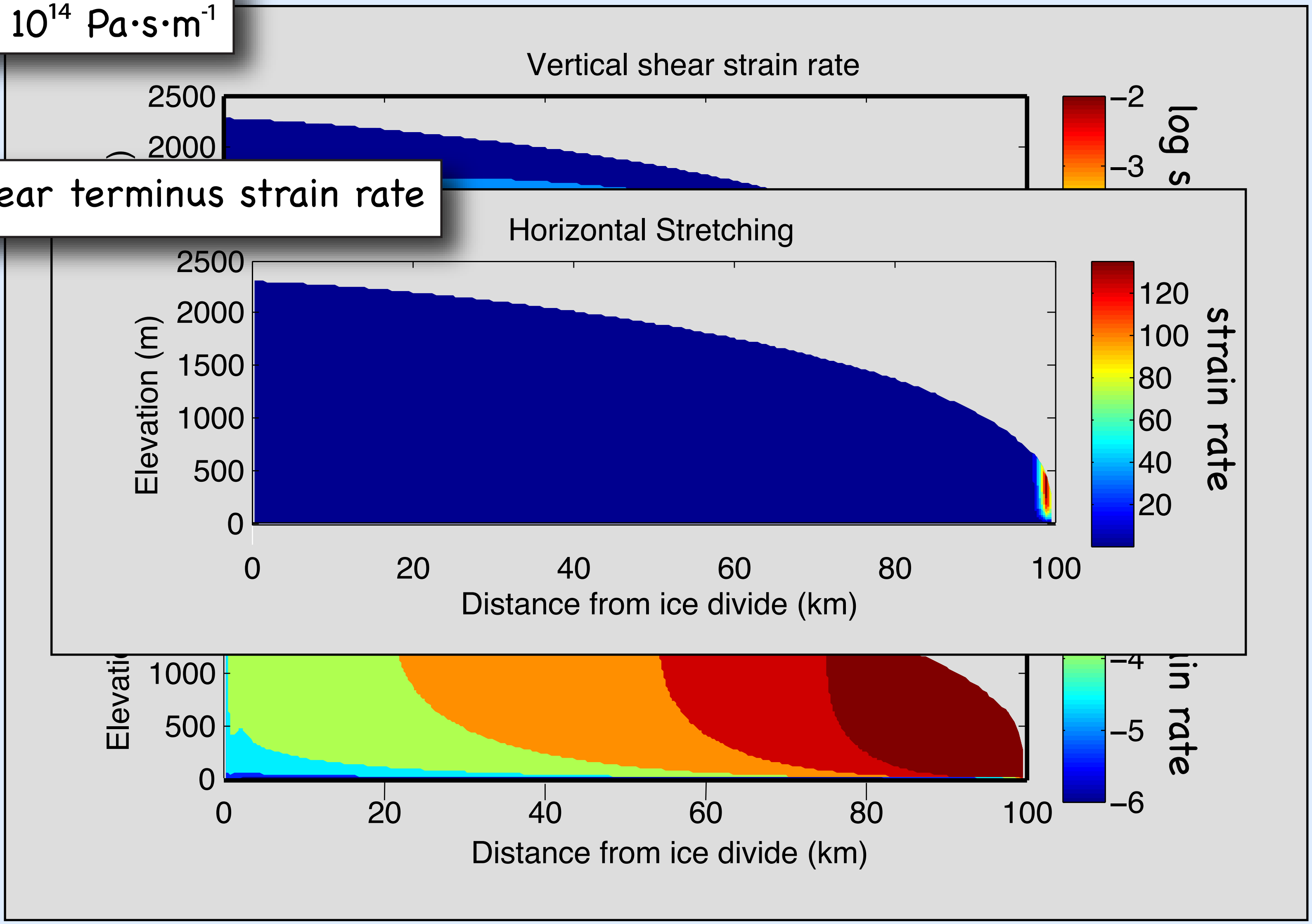

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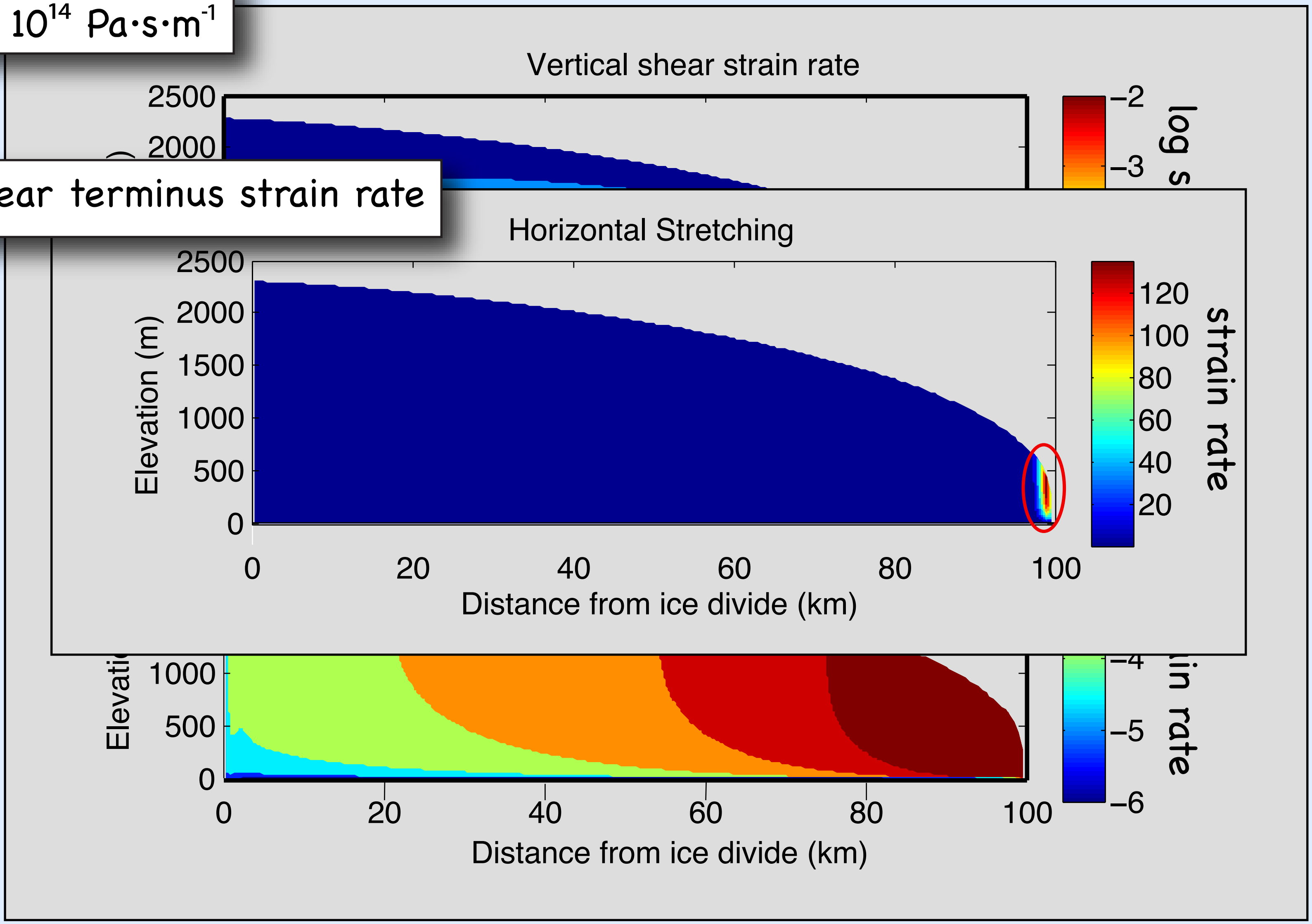

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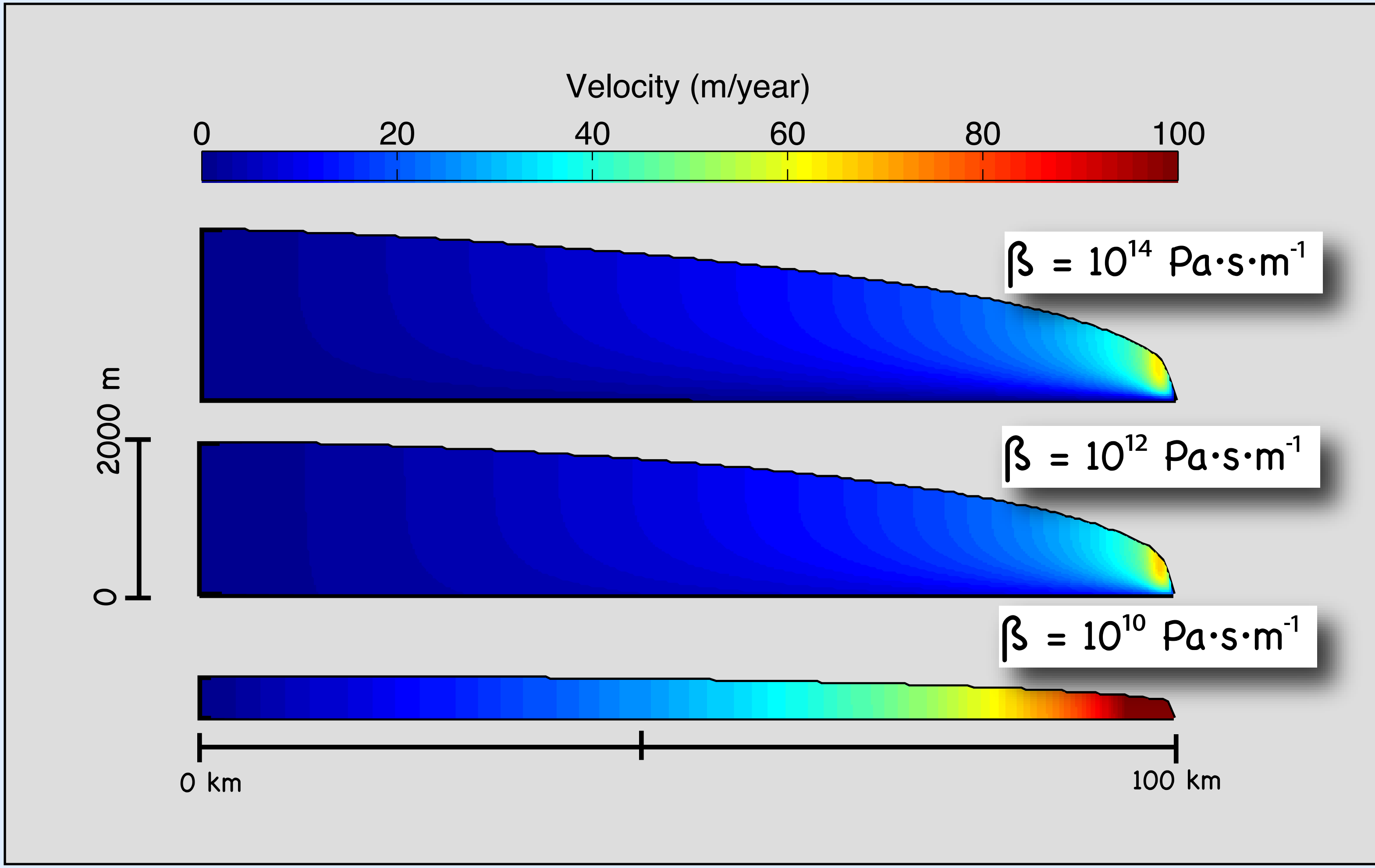

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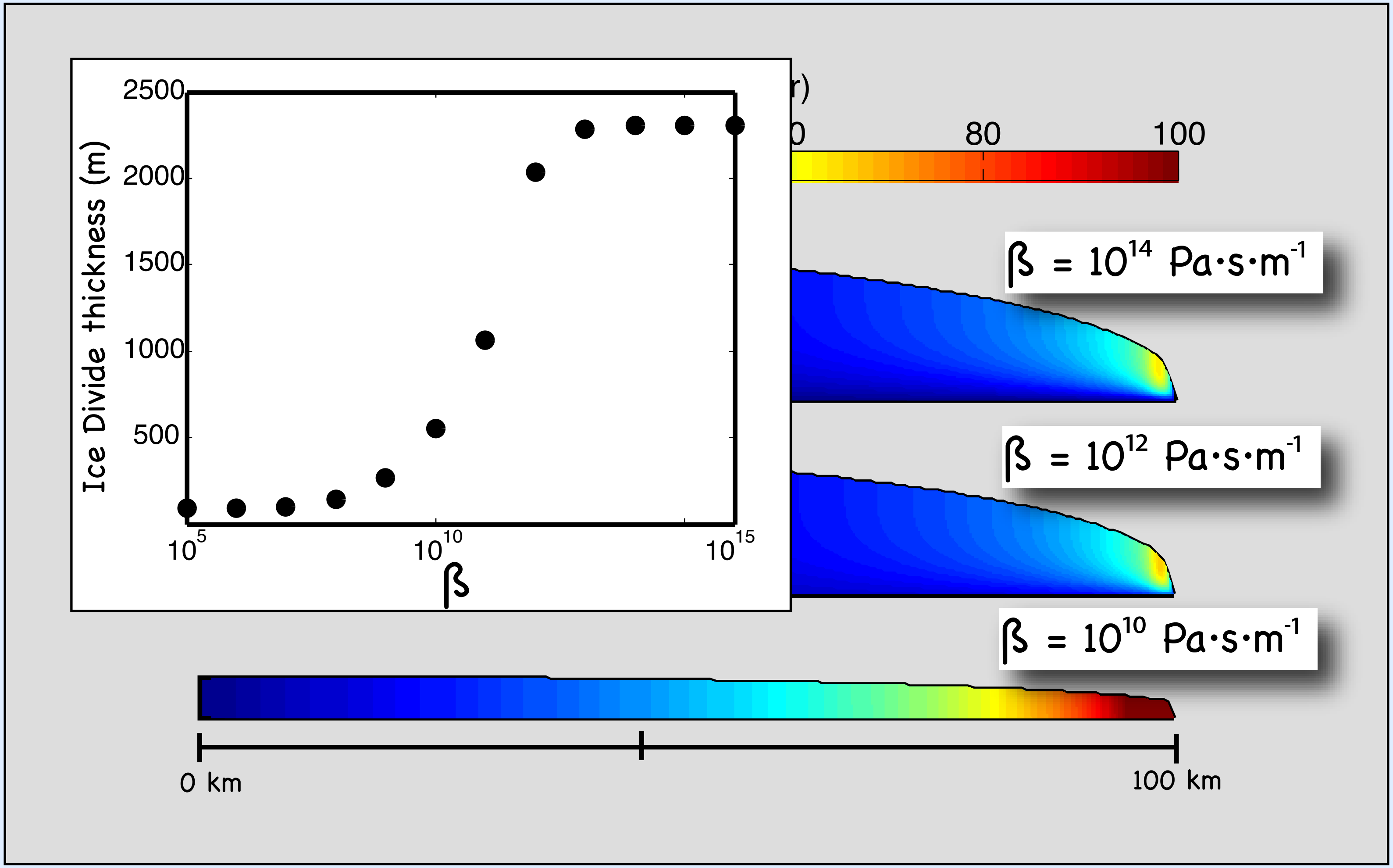

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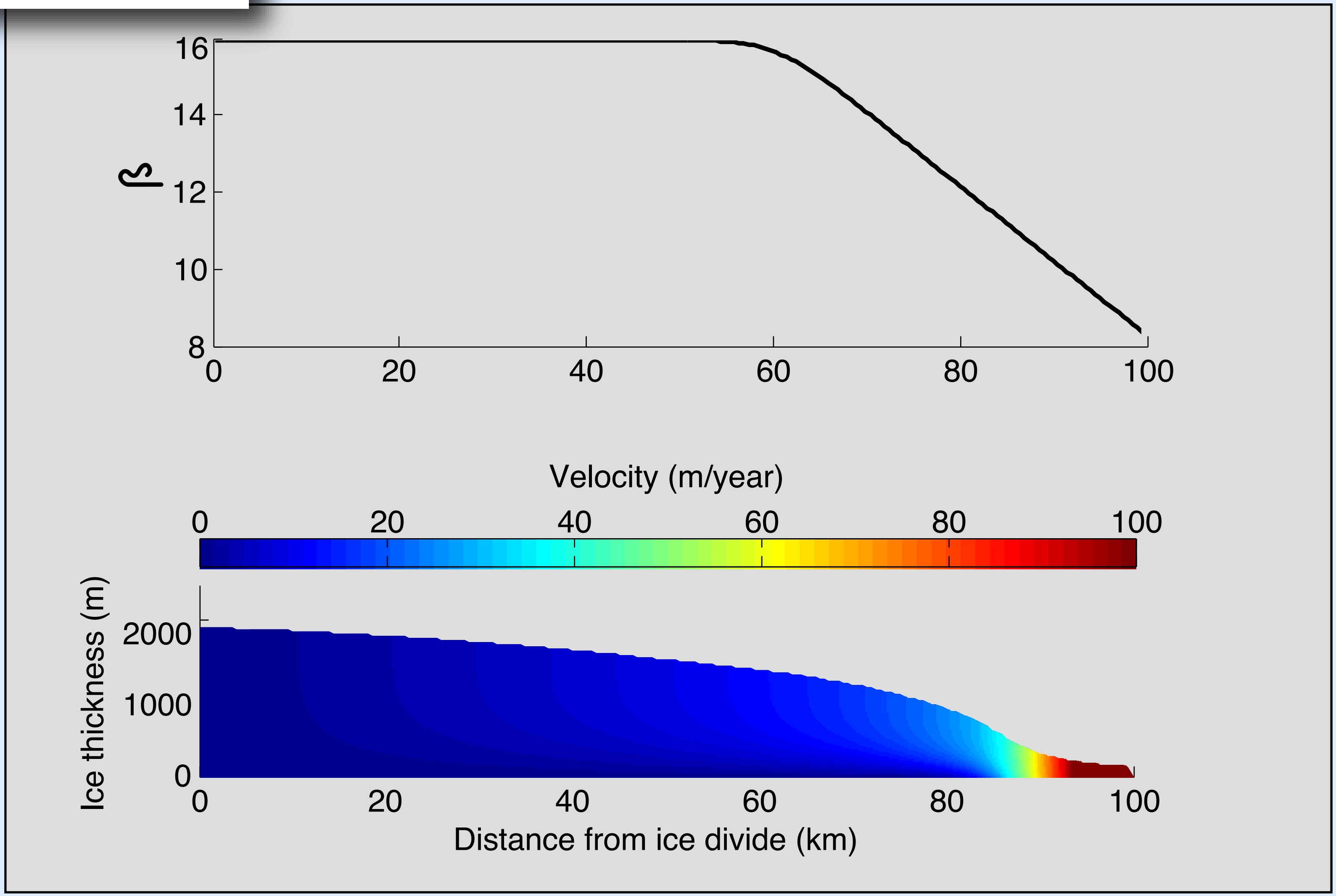

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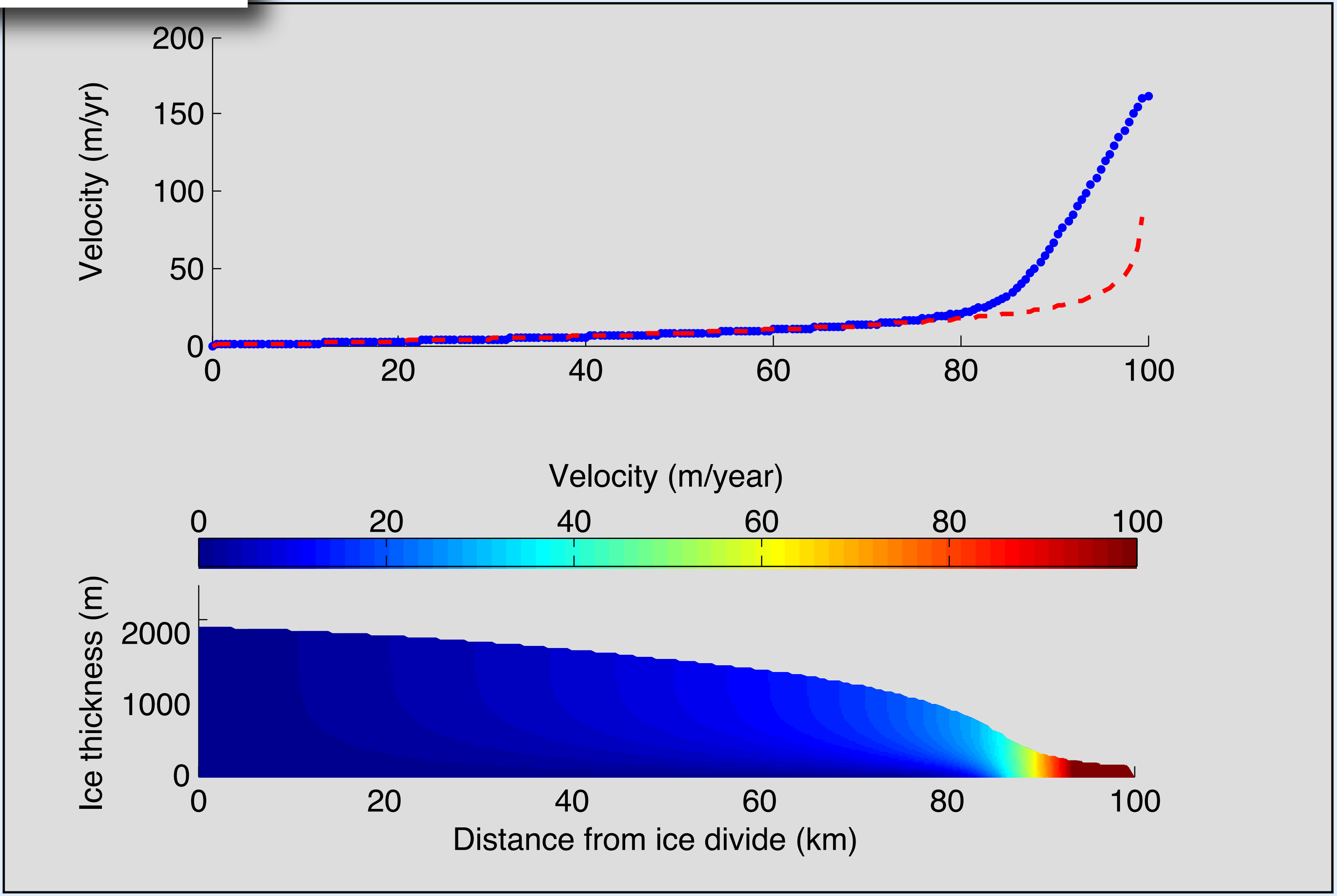

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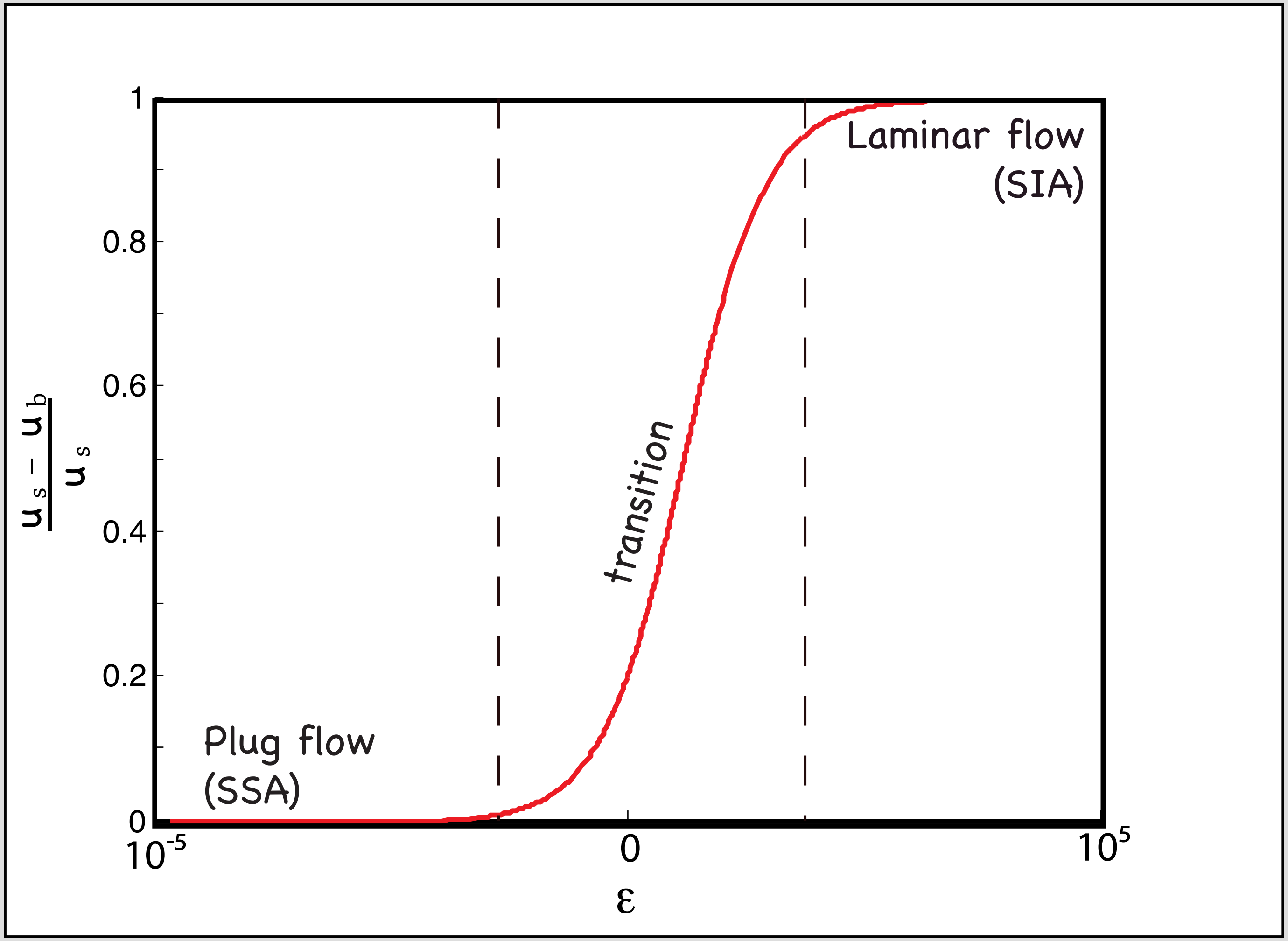

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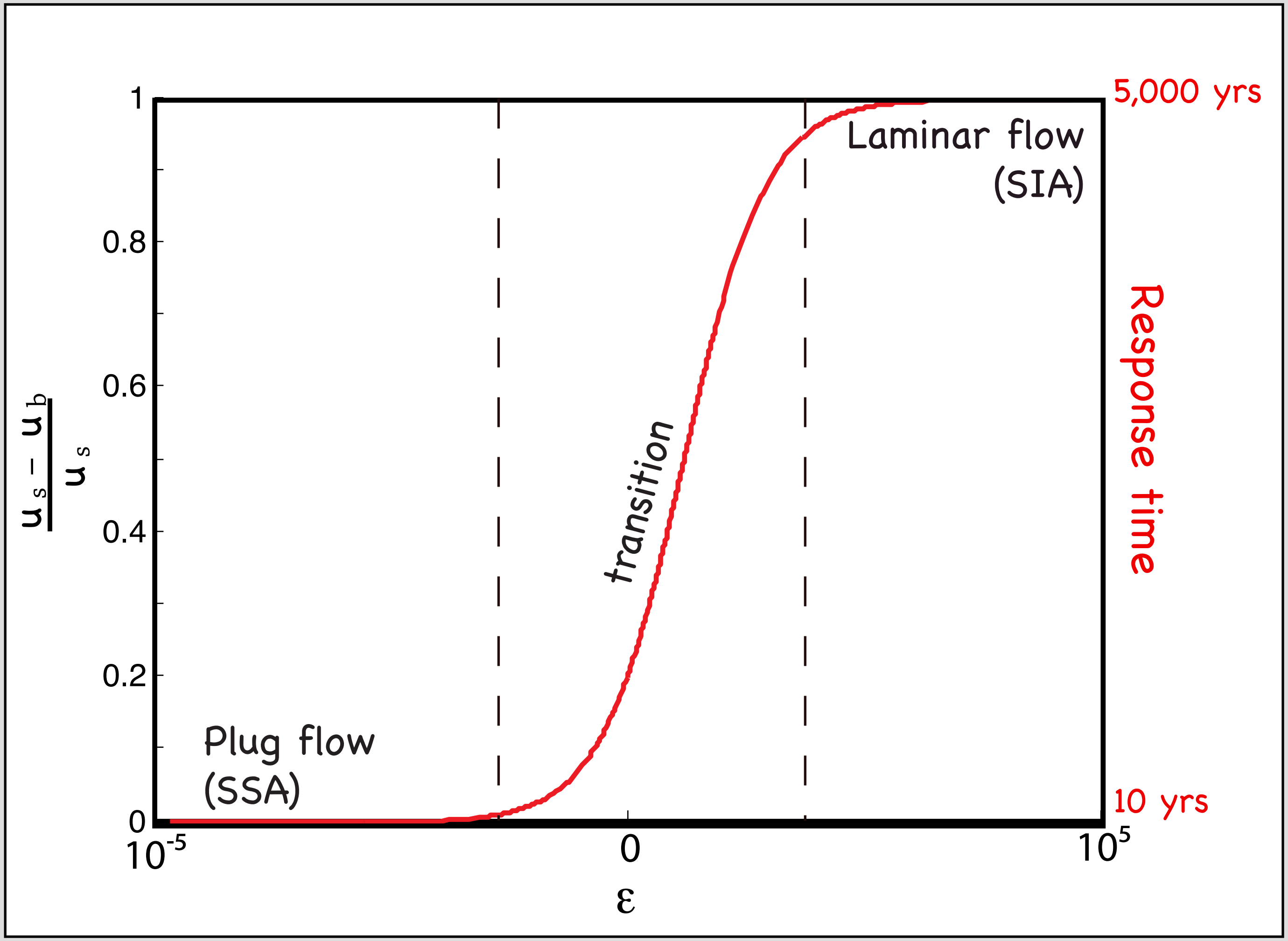

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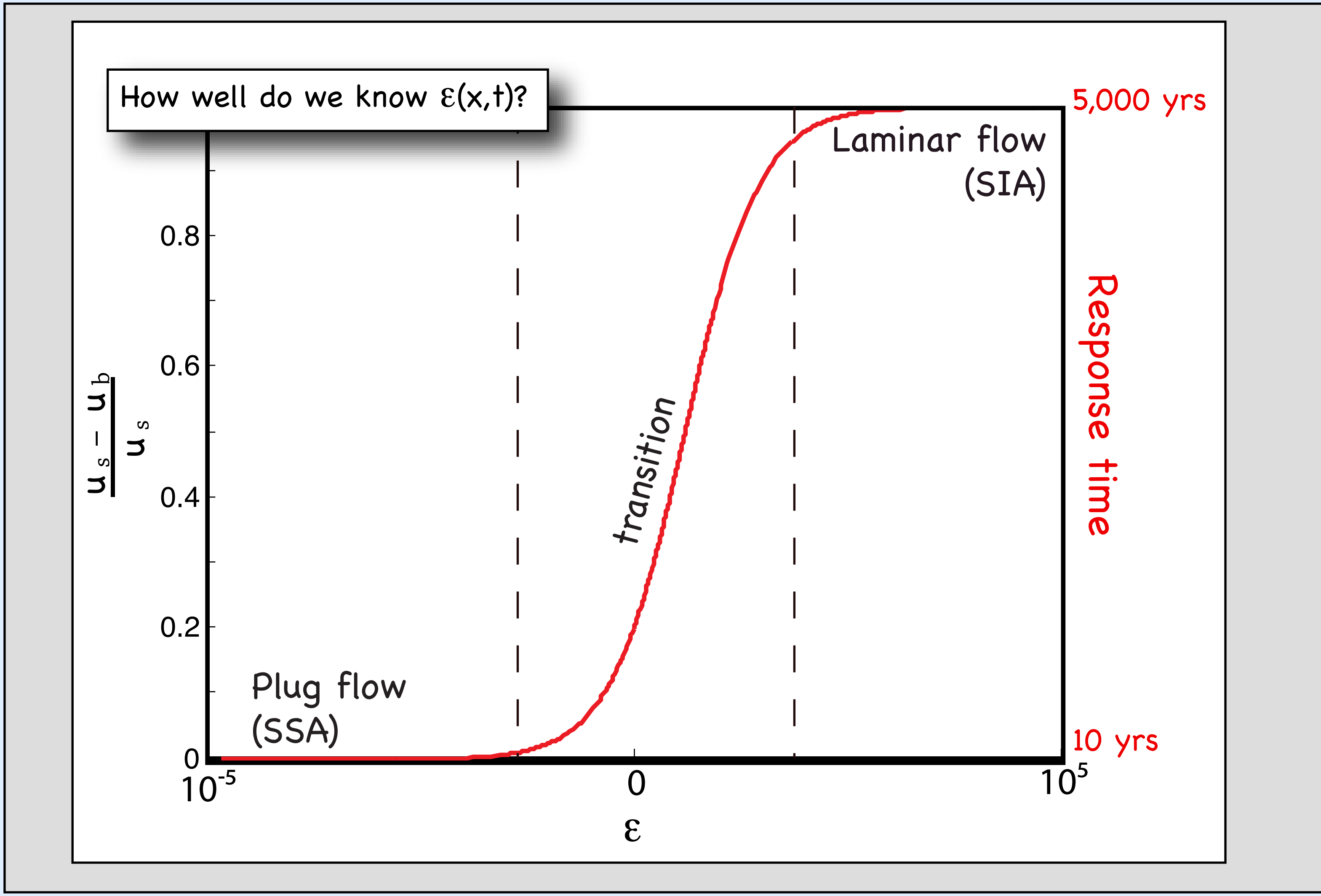

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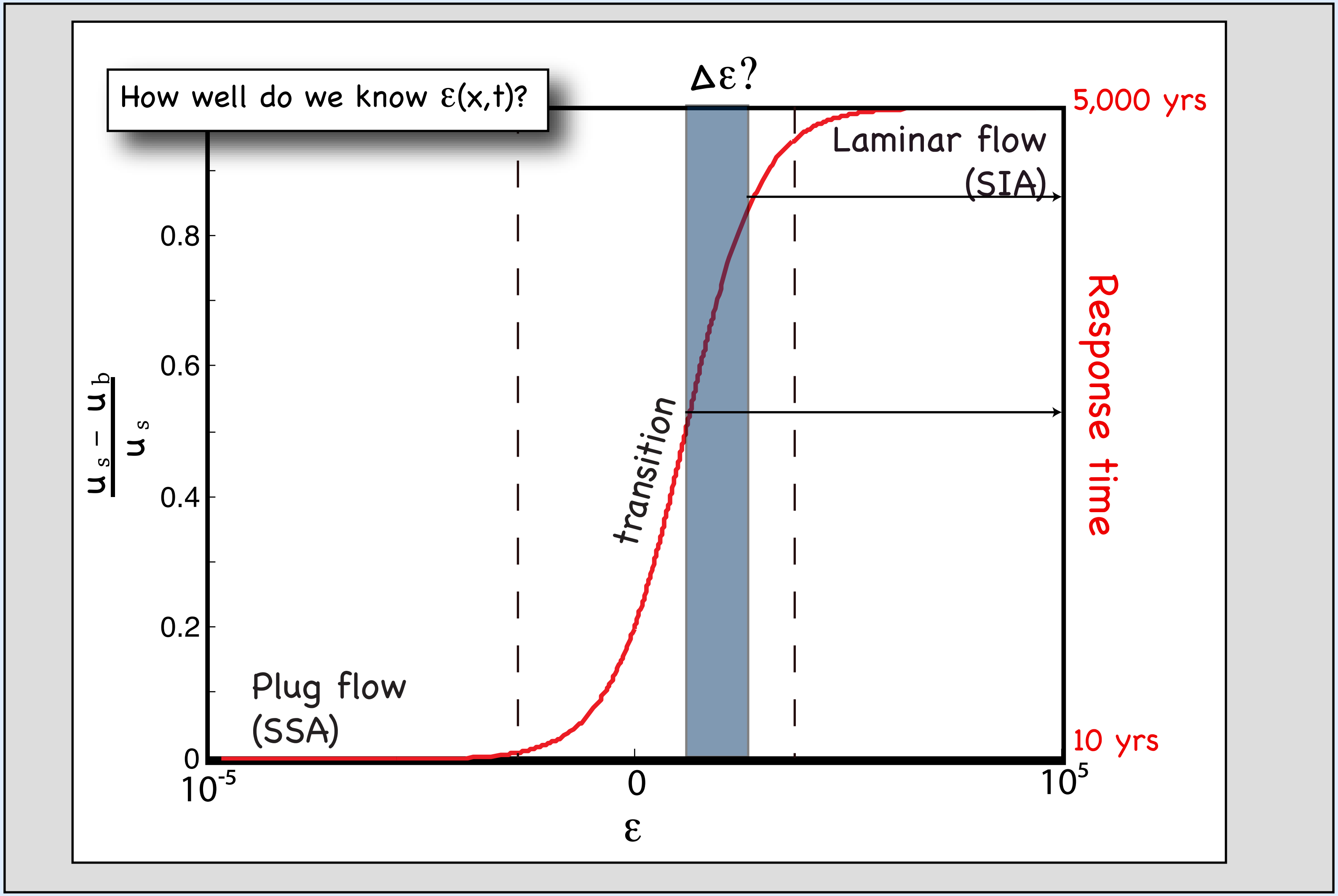

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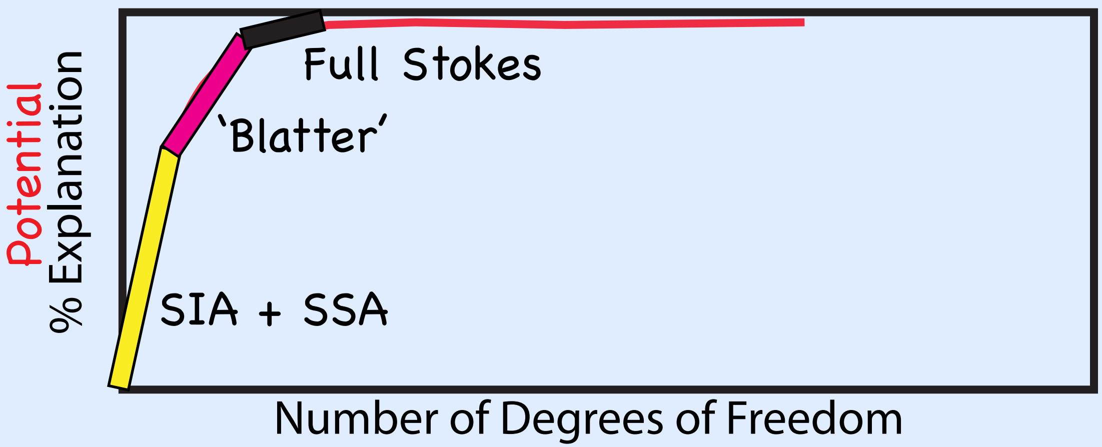

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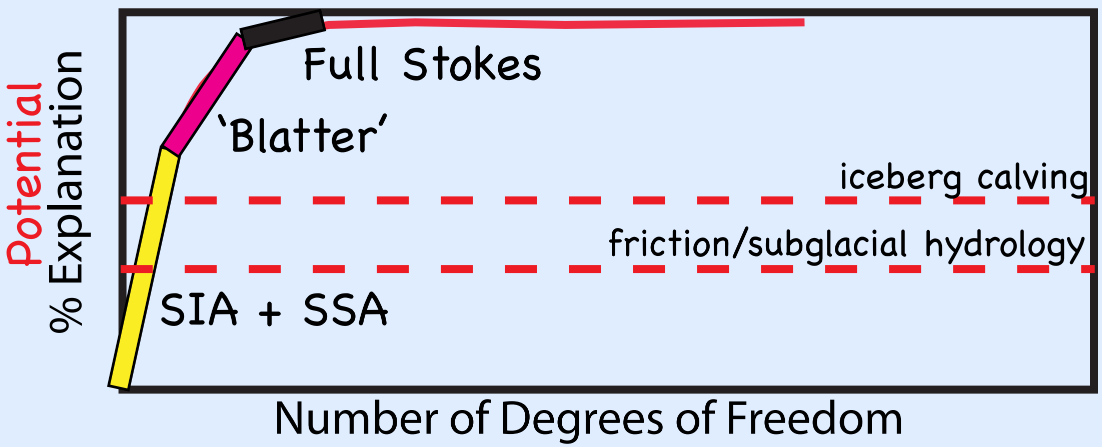

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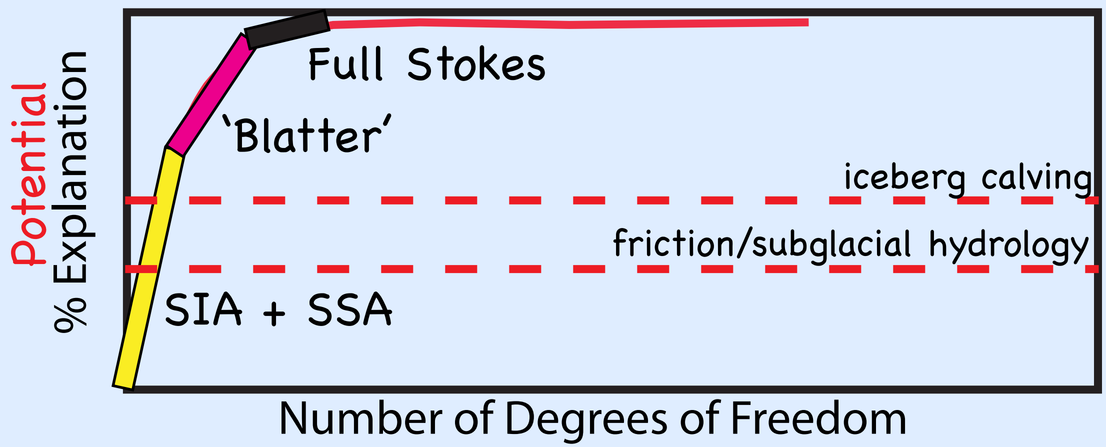

Challenge: Identify and parameterize the 'barrier' processes that limit our understanding

Deux ex machina?

Already developed and in use

`SIA/SSA'
Coarse grids, integration over hundreds of thousands of years

Heavy development now

`Blatter'
Moderate grids, century to thousand year time-scale integration

Highest development cost

`Full Stokes'
High resolution grids, decade to century time-scale integration

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